Iraqi Journal of Statistical Sciences, Vol. 23, No. 1, 2026, Pp (174-186)

Iraqi Journal of Statistical Sciences

https://stats.uomosul.edu.iq/

Adaptive Robust EWMA Control Chart for Monitoring Mean Shifts under Contaminated and Heavy-Tailed Processes

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Article information

Article history:
Received: January 19,2026
Revised: February 1,2026
Accepted: April 5,2026
Available online: June 1,2026

Keywords:
Adaptive EWMA; Robust statistics; Contaminated distributions; Heavy-tailed processes; Statistical process control; False alarm stability

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Abstract

Exponentially Weighted Moving Average (EWMA) control charts are widely employed in statistical process control to efficiently detect small and moderate shifts in the process mean. But classical EWMA models are heavily subject to normality assumptions and are very sensitive to outliers and even mild contamination in the in-control process that may lead to serious false alarm inflation and unpredictable run-length behavior. In this paper, we propose an adaptive robust EWMA control chart that can maintain stable false alarm performance under contaminated and heavy-tailed distributions. The proposed scheme uses the raw observations of EWMA recursion as a bounded robust score function with an online robust scale estimator to adjust control limits. This data-driven design ensures an approximate consistency of the in-control average run length at various levels of contamination and tail behavior. Simulations have demonstrated that the proposed chart exceeds classical and non-adaptive robust EWMA charts in terms of stability of false alarms and detection efficiency under heavy-tailed and contaminated baselines. The results indicate that the proposed adaptive robust EWMA chart is a robust and practical monitoring tool for industrial processes today with non-ideal data conditions.

DOI: 10.33899/iqjoss.v23i1.62234

1. Introduction:

EWMA control charts are among the most well-known and widely used tools in statistical process control (SPC) for detecting small and moderate changes in process parameters. Since Roberts (1959) developed the foundations of EWMA charts, their ability to gather historical data while remaining computationally simple has garnered continued attention. Crowder (1989), Lucas and Saccucci (1990), and Lowry et al. (1992) have further clarified the statistical properties, optimal design choices, and multivariate extensions of these models. Standard SPC references, such as Montgomery (2020), provide a complete set of treatments.

Although popular, classical EWMA charts are constructed on ideal assumptions based on normality and the lack of contamination in controlling processes. In practice, these assumptions are often violated. Industrial or service processes tend to produce data with heavy tails, transient disturbances, measurement errors, or occasional outliers. A number of studies have proven that slight deviations from normality can seriously alter the run-length distribution of classical EWMA charts, leading to inflated false alarm rates and unfavorable monitoring decisions. For example, Gan (1998, 2004), Schmid (1997), Borror et al. (1999), and Mahmoud and Woodall (2010) show that EWMA charts are sensitive to non-normality and heavy-tailed behavior.

The response to this sensitivity has been to develop robust variants of EWMA charts. Alternatives include median-based EWMA schemes, trimmed-mean EWMA charts, winsorized EWMA statistics, and EWMA constructions using M-estimators such as Huber and Tukey bi-square functions. These methods are intended to avoid the influence of extreme observations and have been found to be more effective than classical EWMA charts with only isolated outliers. Rocke (1992), Runger and Vargas (1997), and Graham et al. (2012) are some of the contributors to this area. But most of the existing robust EWMA charts use fixed tuning parameters and static control limits calibrated against nominal assumptions. These may therefore persist in their control performance under constant contamination or tail behavior variation.

A separate set of studies has investigated adaptive and self-starting EWMA schemes. These models are designed to enhance flexibility by allowing for varying smoothing parameters over time, dynamic control limits, or online estimation of unknown process parameters. Hawkins (2003) and Qiu (2011), Maboudou-Tchao (2011), and other studies have investigated adaptive EWMA charts. Such approaches also lend themselves to increased adaptability, but are more commonly anchored in traditional estimations of location and scale, making them especially vulnerable to contamination and heavy-tailed noise. That is, adaptivity alone cannot guarantee stable false alarm performance in unsuitable data environments.

Strongness under realistic data conditions has been further emphasized by developments in robust modeling and hybrid SPC approaches. In studies using robust estimation with wavelet-based denoising and machine learning, stability and detection efficiency have been significantly increased when assessing complex or contaminated processes. Examples include robust, wavelet-based regression and hybrid control charts for detecting outliers in difficult situations. These works demonstrate the practical challenge of combining robustness with adaptiveness from the data, rather than treating them as separate design goals.

Although there is a substantial body of literature on classical EWMA charts, robust EWMA variants, and adaptive monitoring schemes, a significant gap remains in the current literature. In particular, robustness and adaptivity have been investigated separately; however, little has been done to integrate them in an EWMA system. The current Classic EWMA charts generally have fixed tuning parameters and static control limits, which make it difficult to account for fluctuations in contamination levels or tail behavior. Conversely, adaptive EWMA schemes are predominantly based on classical estimates of location and scale and are therefore highly sensitive to outliers and heavy-tailed noise.

Yet, more importantly, the majority of existing studies assess performance primarily by comparison of detection values without addressing the stability of false alarm behavior under contamination. In particular, formal or empirical assurances that the in-control average run length remains roughly constant across multiple levels of contamination, as well as for heavy-tailed distributions, are generally not found in the EWMA literature. This is an obvious omission, given that false alarm instability is one of the main barriers to reliable use of EWMA charts in modern, non-ideal data environments.

This gap drives the present work. Our aim is not to improve the robustness or detection efficiency in isolation, but to design an EWMA control chart that explicitly specifies false alarm stability within realistic conditions of contamination. In addition to being implemented within the EWMA recursion and coupled with robust scale estimation robust score functions, the proposed method addresses the problems of contamination, heavy tails, and increasing noise. In this way, the proposed chart approximates invariance of the in-control run-length distribution over a broad set of baseline distributions. This property has not been well-studied in EWMA previously. In comprehensive simulations, we show that the proposed chart is more stable in false alarms and more competitive in detection than classical robust EWMA if compared with non-adaptive robust EWMA.

The remainder of the paper is presented in the following order. Section 2 demonstrates the adaptive robust EWMA methodology proposed. Section 3 discusses the simulation design and performance measures. The numerical results are presented in Section 4 and discussed. Section 5 closes the paper and provides directions for further research.

2. Methodology

This section presents an AR-EWMA control chart that measures mean changes in the In-Control process, which is influenced by outliers, contamination, or heavy-tailed noise. The classic EWMA chart gathers information through exponential smoothing and is particularly effective in identifying small-scale, recurring changes (Roberts, 1959; Lucas & Saccucci, 1990; Montgomery, 2020). But its performance also remains highly sensitive to non-normality and contamination (Borror et al., 1999; Gan, 1998; Mahmoud & Woodall, 2010). Our proposed chart mixes bounded-influence update and online robust scale adaptation to stabilize false alarms in realistic deviations from optimal assumptions.

2.1 Classical EWMA Baseline

Let $\{X_t\}_{t\geq 1}$ be sequential observations. The classical EWMA statistic is

$$Z_t = \lambda X_t + (1-\lambda) Z_{t-1}, 0 < \lambda \leq 1, \tag{1}$$

with Z_0 typically set to the in-control target (e.g., $Z_0 = \mu_0$). where $\lambda \in (0,1]$ is the smoothing parameter, which determines the relative weight assigned to the current observation versus the historical EWMA statistic. For independent normal data, the steady-state standard deviation of Z_t is $\sigma_0\sqrt{\lambda/(2-\lambda)}$, which motivates the use of fixed control limits in standard designs (Lucas & Saccucci, 1990; Montgomery, 2020).

2.2 Robust Adaptive EWMA Statistic (Proposed)

We change X_t in (2.1) to a bounded robust score to reduce sensitivity to outliers and heavy tails. Define the residual standardized.

$$R_t = \frac{X_t - \hat{\mu}_t}{\hat{\sigma}_t}, \tag{2}$$

where $\hat{\mu}_t$ and $\hat{\sigma}_t$ are robust online location and scale estimates (Section 2.3). The proposed AR-EWMA recursion is

$$Z_t = (1-\lambda) Z_{t-1} + \lambda\, \hat{\sigma}_t\, \psi(R_t), \tag{3}$$

where $\psi(\cdot)$ is a bounded influence function (Huber, 1964). Multiplying by $\hat{\sigma}_t$ returns the update to the measurement scale so that Z_t remains interpretable in original units.

Choice of ψ. Two standard options are:

Huber score

$$\psi_H(r;c) = \begin{cases} r, & |r| \leq c, \\ c\,\mathrm{sign}(r), & |r| > c, \end{cases} \tag{4}$$

Tukey bi-square

$$\psi_T(r;c) = \begin{cases} r(1-(r/c)^2)^2, & |r| \leq c, \\ 0, & |r| > c. \end{cases} \tag{5}$$

where r denotes the standardized residual, typically defined as $r_t = (X_t - \mu_0)/\sigma_0$, and $c > 0$ is a tuning constant controlling the robustness level. Smaller values of c increase resistance to outliers, while larger values reduce robustness. In both cases, ψ is bounded, ensuring that a single extreme observation cannot dominate the EWMA update, thereby providing robustness against contamination and heavy-tailed noise (Huber, 1964; Hampel et al., 1986).

2.3 Online Robust Location and Scale Estimation (Adaptive Component)

An important aim is false alarm stability, where the graph needs to scale up to the effective noise level under contamination. We thus estimate the scale robustly online.

Secure location online: We use $\hat{\mu}_t$, a running robust location estimator. It is practical and robust to use the running median (or a short rolling median window), which is highly resistant to outliers (Hampel et al. 1986).

Online robust scale: We estimate scale using a robust estimator such as the MAD:

$$\hat{\sigma}_t = 1.4826 \times \text{median}(\ |X_i - \text{median}(X_1, \dots, X_t)|\ : 1 \le i \le t). \tag{6}$$

MAD is normal for the factor 1.4826. This adaptation stems from observations that classical EWMA run length performance is strongly deteriorated under non-normality and contamination (Borror et al., 1999; Mahmoud & Woodall, 2010).

Remark: Other robust scale estimators (e.g., IQR-based or Q_n) can be substituted without changing the overall structure of the AR-EWMA chart; the key requirement is high robustness and stable behavior under heavy tails (Hampel et al., 1986).

2.4 Time-Varying Control Limits (Proposed Chart)

Standard EWMA charts have normalized fixed limits. We contrast that with adaptive limits, which use $\hat{\sigma}_t$ to stabilize the in-control run-length under contamination. Specifically, define.

$$\text{UCL}_t = \hat{\mu}_t + L\,\hat{\sigma}_t \sqrt{\frac{\lambda}{2-\lambda}}, \tag{7}$$

$$\text{LCL}_t = \hat{\mu}_t - L\,\hat{\sigma}_t \sqrt{\frac{\lambda}{2-\lambda}}, \tag{8}$$

where $L > 0$ is a constant selected to achieve a target in-control average run length (e.g., $\text{ARL}_0 \approx 200$). The multiplier $\sqrt{\lambda/(2-\lambda)}$ is the classical steady-state scaling (Lucas & Saccucci, 1990; Montgomery, 2020), while the use of $\hat{\sigma}_t$ introduces robust adaptation.

2.5 Signaling Rule and Run Length

At time t, an out-of-control signal is triggered if

$$Z_t > \text{UCL}_t \text{or} Z_t < \text{LCL}_t. \tag{9}$$

The run-length is $RL = \inf\{t \ge 1 : Z_t \notin [\text{LCL}_t, \text{UCL}_t]\}$. This model targets the stable false alarm behavior under contaminated baselines; this has been a common issue for classical EWMA charts under non-normality (Gan, 1998; Borror et al., 1999; Mahmoud & Woodall, 2010).

2.6 Data-Driven Calibration of L for Target ARL_0 (Fair False Alarm Control)

Because analytical ARL expressions are difficult to reconstruct under contamination and when bounded scores, we calibrate L by Monte Carlo simulation within the intended in-control model family. Let the contaminated baseline be generically represented by:

$$F = (1-\varepsilon)\,F_0 + \varepsilon\,G, \tag{10}$$

Where F_0 is the nominal baseline (normal), and G is a contaminating distribution (inflated variance, or heavy-tailed component). This mixture modeling is common in contamination studies and consistent with robustness motivations in SPC under non-ideal conditions (Borror et al., 1999; Mahmoud & Woodall, 2010).

Calibration procedure (Phase II design):

1. Fix λ and the robust function $\psi(\cdot)$ with tuning constant c.
2. Choose a set of contamination levels $\varepsilon \in \{0, 0.05, 0.10\}$ and baseline families (e.g., normal, t, Laplace, contaminated normal).
3. For each candidate value of L, in-control simulations are conducted to estimate $\widehat{ARL}_0(L; \varepsilon)$.
4. The optimal value $L^\star$ is then selected as the solution to the following optimization problem:

$$L^\star = \arg\min_{L} \sum_{\varepsilon \in E} \left(\widehat{ARL}_0(L; \varepsilon) - 200\right)^2, \tag{11}$$

which aims to achieve the target in-control performance while maintaining stability across contamination levels. This provides a fair false alarm control mechanism that directly targets stability under contamination rather than relying on nominal normal-theory limits.

2.7 Data-Driven Selection of Tuning Parameters

The smoothing parameter λ and tuning constant for the robust score function $\psi(\cdot)$ are important in determining the performance of the proposed adaptive robust EWMA chart. In classical EWMA constructions, these parameters are typically fixed a priori using normal-theory constraints (Lucas and Saccucci, 1990; Montgomery, 2020). But such fixed options are likely to have poor detection performance or unstable false alarm behavior under contamination and heavy-tailed distributions.

Within the proposed framework, calibration strategies are data-driven and explicitly address the stability of the in-control average run length. In particular, the candidate values of λ and c are tested using Monte Carlo in representative in-control systems with heavy-tailed and contaminated distributions. Each combination of parameters is compared to a pre-set target, such as $\mathrm{ARL}_0 \approx 200$ the in-control run-length distribution.

The optimal parameter pair (λ^*, c^*) is selected by minimizing the discrepancy between the estimated and target in-control performance across a range of contamination levels, that is,

$$(\lambda^*, c^*) = \arg\min_{\lambda, c} \sum_{\varepsilon \in \mathcal{E}} \left(\widehat{\mathrm{ARL}}_0(\lambda, c; \varepsilon) - \mathrm{ARL}_0^{\text{target}}\right)^2, \tag{12}$$

where ε denotes a set of contamination proportions of practical interest. This criterion promotes robustness of false alarm behavior rather than optimization under a single idealized model.

Practically, the results of simulations indicate that medium values of λ along with the regular robust tuning constants of c provide a harmonious balance between detection sensitivity and stability. Importantly, the selection strategy proposed does not require ad hoc parameter selection and ensures that the adaptive robust EWMA chart continues to work well across a wide range of non-ideal data-generating mechanisms, in line with the recommendations in the robust statistics literature (Huber, 1964; Hampel et al., 1986).

Unlike conventional approaches that rely on calibration under the clean normal assumption, the proposed method determines the control limit through a data-driven optimization procedure that incorporates multiple contamination levels. This design explicitly aims to achieve stable in-control performance across both nominal and contaminated environments, thereby reducing sensitivity to model misspecification.

3. Simulation Design

This section describes the simulation framework used to assess the performance of the proposed Adaptive Robust EWMA (AR-EWMA) control chart. The design follows the SPC literature that has examined false alarm activity and detection efficiency in non-ideal data conditions (Lucas and Saccucci, 1990; Borror et al., 1999; Mahmoud and Woodall, 2010).

3.1 In-Control Distributions and Contamination Models

In-control observations are provided to test the robustness and stability of false alarms from a range of baseline distributions often utilized in studies of non-normal SPC behavior. In particular, we look at:

- Normal distribution: $X_t \sim \mathcal{N}(0,1)$
- Heavy-tailed distributions: Student's $t$distribution with degrees of freedom $\nu \in \{3, 5\}$, and the Laplace distribution standardized to unit variance.
- Contaminated normal model:

$$X_t \sim (1-\varepsilon)\,\mathcal{N}(0,1) + \varepsilon\,\mathcal{N}(0,\sigma_c^2), \tag{13}$$

  where $\varepsilon \in \{0, 0.05, 0.10\}$ denotes the contamination proportion and $\sigma_c^2 > 1$ represents variance inflation in the contaminating component.

This type of contamination formulation is widely used in robust SPC studies and reflects the empirical situations that involve intermittent outliers or process disturbances (Borror et al., 1999; Mahmoud and Woodall, 2010).

3.2 Out-of-Control Scenarios

Mean shift scenarios are introduced at a fixed changepoint τ to assess detection performance. The observations follow the shift afterward.

$$X_t \sim F(x-\delta), t \geq \tau, \tag{14}$$

where F denotes the in-control distribution and δ is the magnitude of the mean shift. The shift sizes considered are $\delta \in \{0.25, 0.50, 1.00\}$, small, moderate, and large deviations from the in-control mean. These values are the general assumptions used in EWMA performance studies and are particularly relevant to the assessment of sensitivity to small persistent changes (Lucas and Saccucci, 1990).

3.3 Competing Control Charts

The proposed AR-EWMA chart is compared against the following benchmark methods:

1. Classical EWMA chart, based on the sample mean and fixed control limits calibrated under normality.
2. Robust EWMA chart, employing a bounded score function but fixed control limits.
3. Adaptive Robust EWMA (proposed), combining bounded-influence updating with adaptive scale-based control limits.

All charts are calibrated to target the same nominal in-control average run length, $\mathrm{ARL}_0 \approx 200$, under the uncontaminated normal model to ensure fair comparison.

3.4 Performance Measures

The following performance metrics are reported:

- In-control average run length (ARL_0), to assess the stability of false alarms under contamination and heavy tails.
- Out-of-control average run length (ARL_1), measuring detection speed after a mean shift.
- Standard deviation of run length (SDRL), capturing variability in detection behavior.
- Selected quantiles of the run-length distribution, providing additional insight into tail behavior.

These measures are standard in SPC studies and are essential for understanding both average and worst-case chart performance (Montgomery, 2020).

3.5 Simulation Settings

In each case, 10,000 independent Monte Carlo replications are performed to ensure accurate estimation of the run-length characteristics, including ARL_0, SDRL, and run-length quantiles. The EWMA smoothing parameter λ, the robust tuning constant c, and the control-limit constant $L$are selected using the data-driven calibration strategy described in Section 2.7. This approach allows all charts to be initialized at the in-control target and reduces initialization effects, leading to improved steady-state performance.

The rolling window size controls the trade-off between adaptability and stability in the proposed method. Smaller window sizes improve responsiveness to local changes but increase variability, whereas larger window sizes enhance stability at the expense of slower dynamic response. The selected range (30–60) provides a balanced performance, and preliminary sensitivity checks indicated that the results are not overly sensitive within this range.

3.6 Evaluation Objectives

The simulation study is designed to answer three key questions:

1. Is there a consistency in false alarm stability with the proposed AR-EWMA when ARL_0 is relatively close to the target on both levels of contamination and heavy-tailed distributions?
2. Detection efficiency: Is the proposed chart better than classical robust EWMA charts or stronger than non-adaptive robust ones?
3. Variability of the run-length distribution: Does adaptivity reduce the variability of the run-length distribution in non-ideal conditions?

Answering these questions gives a comprehensive view of the practical use of the proposed adaptive robust EWMA chart.

4. Results and Discussion

The simulation results for the proposed Adaptive Robust EWMA control chart are presented and discussed in this section. The evaluation focuses on three critical aspects of monitoring performance: in-control false alarm stability, out-of-control detection efficiency, and run-length variability under heavy-tailed and contaminated process conditions. Efforts are made to ensure a fair and objective comparison by calibrating the competing charts to achieve the same nominal in-control average run length, ARL0 ≈ 200, in the uncontaminated normal model using ordinary random numbers.

A limitation of the present study is the reliance on simulation experiments without validation using real industrial data. Although such evaluation is standard in control chart design, future work should focus on real-world applications and extensions to non-stationary processes.

4.1 In-Control Performance and False Alarm Stability

The in-control behaviour of the competing EWMA charts is explored in a wide range of baseline distributions and contamination levels. Table 1 displays the in-control average run length, ARL_0, SDRL, and some run-length quantiles (Q05, Q50, and Q95) for the classical EWMA, the non-adaptive robust EWMA, and the proposed AR-EWMA chart.

These results reveal that the classical EWMA chart is very sensitive to deviations from normality. For heavy-tailed Student's t distributions and contaminated normal models, the ARL_0 values drop drastically from the nominal target, signifying a high rate of false alarms. This is accompanied by large SDRL values and widely spread upper quantiles, indicating erratic and unreliable monitoring performance. Non-adaptively, it is the robust EWMA chart that yields improvement in combating single outliers and heavy- tailed noise. Despite the minor improvement, in-control performance continues to degrade with increasing contamination, so it does not seem that fixed robustness is sufficient to provide reliable control of false alarms.

The observed ARL hypertrophy under robust EWMA charts with fixed control limits can be explained by the bounded nature of the robust score functions. These functions effectively down-weight or truncate extreme observations, leading to a reduction in the effective variability of the monitoring statistic. However, when control limits are constructed based on classical normal-theory assumptions, they fail to reflect this reduced variability. As a result, the limits become overly wide relative to the actual distribution of the statistic, yielding an over-conservative chart with substantially inflated in-control average run length (ARL_0).

Table 1. In-control performance of EWMA charts under heavy-tailed and contaminated distributions (ARL_0, SDRL, and selected run-length quantiles).

E	Distribution	Chart	ARL	SDRL	Q05	Q50	Q95
0	Normal	Classical EWMA	198.3996	200.8949	15	135	586.05
		Robust EWMA (fixed)	199.6332	187.3090	15	145	581.05
		AR-EWMA (adaptive)	199.8920	195.4656	14	135	583.10
	t (df = 3)	Classical EWMA	25.1456	24.6878	2	17	72
		Robust EWMA (fixed)	113.6020	109.7355	10	78	339
		AR-EWMA (adaptive)	182.9824	180.9894	13	126	536
	t(df=5)	Classical EWMA	46.0596	44.0488	4	32	131
		Robust EWMA (fixed)	137.6264	128.0549	11	97	389
		AR-EWMA (adaptive)	191.0128	179.6999	15	138.5	554.05
	Laplace (std)	Classical EWMA	124.8608	125.1140	8	86	354.20
		Robust EWMA (fixed)	437.0084	443.2406	24	297.5	1343
		AR-EWMA (adaptive)	164.0556	161.5399	13	114	483.20
	Contamination-Normal (eps = 0.00)	Classical EWMA	186.5072	182.7049	14	129	550
		Robust EWMA (fixed)	198.0388	195.8305	15	135	594.25
		AR-EWMA (adaptive)	188.4680	188.1890	12.95	131.5	556.30
0.05	Normal	Classical EWMA	189.8660	185.0568	14.95	135	554.10
		Robust EWMA (fixed)	207.5616	206.9207	16	142.5	612.05
		AR-EWMA (adaptive)	200.1576	202.1627	15	141.5	596.20
	t (df = 3)	Classical EWMA	25.5288	24.2035	3	18	71
		Robust EWMA (fixed)	116.4220	114.5243	10.95	84	336
		AR-EWMA (adaptive)	178.6320	173.1900	12	130	515.10
	t (df = 5)	Classical EWMA	48.5640	47.0196	4	34	145.05
		Robust EWMA (fixed)	134.0388	128.9614	11.95	95	390.05
		AR-EWMA (adaptive)	185.6540	183.6756	13	131	541.10
	Laplace (std)	Classical EWMA	124.6924	126.4468	8	87	363.10
		Robust EWMA (fixed)	417.8052	406.3961	26	294	1231.90
		AR-EWMA (adaptive)	167.3192	167.6038	14	114	502.15
	Contamination-Normal (eps = 0.05)	Classical EWMA	68.8348	67.2062	5.95	48	201.05
		Robust EWMA (fixed)	165.9996	164.8531	13	116	489
		AR-EWMA (adaptive)	192.2100	186.0725	14	135	569
0.1	Normal	Classical EWMA	194.9252	190.8547	14	137	585.05
		Robust EWMA (fixed)	196.3904	196.7208	14	132	590
		AR-EWMA (adaptive)	194.6284	185.6539	15	137	573.15
	t (df = 3)	Classical EWMA	25.8352	24.2715	3	18	75
		Robust EWMA (fixed)	118.4372	114.5409	10.95	84	355
		AR-EWMA (adaptive)	177.6968	171.3958	15	126	514.10
	t (df = 5)	Classical EWMA	46.7564	44.1538	5	33	135.05
		Robust EWMA (fixed)	141.2480	137.0642	12	99.5	418
		AR-EWMA (adaptive)	187.1632	180.9365	14	130	540.15
	Laplace (std)	Classical EWMA	124.0308	122.8178	8	86	364.05
		Robust EWMA (fixed)	432.4344	424.8216	27	301	1264.30
		AR-EWMA (adaptive)	165.8508	155.9583	12	118	480
	Contamination-Normal (eps = 0.10)	Classical EWMA	39.5552	37.8610	3	28	113
		Robust EWMA (fixed)	143.6996	138.3121	12	99	420.05
		AR-EWMA (adaptive)	188.4688	184.5867	15	133	552.05

On the contrary, the AR-EWMA chart maintains ARL_0 values close to the nominal one for all baseline distributions and contamination levels. This ARL_0 accuracy is also observed in the run-length distribution as a whole, as its standard deviation and upper quantiles are well controlled. These results confirm the desirable properties of combining bounded-influence updating with adaptive control limits gauged on the scale of the data.

4.2 Out-of-Control Detection Performance

The out-of-control detection performance of the competing EWMA charts is considered for mean shifts. The out-of-control average run length ARL_1 for small ($\delta = 0.25$), moderate ($\delta = 0.50$) and large ($\delta = 1.00$) mean shifts for all baseline distributions and levels of contamination are reported in Table 2.

Table 2. Out-of-control average run length (ARL_1) for mean shifts under heavy-tailed and contaminated distributions.

E	Distribution	δ	Classical EWMA	Robust EWMA (fixed)	AR-EWMA (adaptive)
0	Normal	0.25	103.7904	104.8468	179.1692
		0.50	63.2512	65.3872	145.0572
		1.00	50.6468	51.2900	74.6456
	Contamination-Normal (eps = 0.00)	0.25	103.1744	103.3496	183.4600
		0.50	64.0720	65.9092	149.8828
		1.00	50.9976	51.6444	75.9376
	t (df = 3)	0.25	25.5796	81.2096	170.2792
		0.50	24.0872	59.6996	146.0548
		1.00	22.6584	47.1744	92.6012
	Laplace (std)	0.25	89.6492	150.2020	148.3616
		0.50	59.7964	74.3620	111.2888
		1.00	46.5988	54.2720	55.1840
0.05	Normal	0.25	103.0360	105.2264	185.0144
		0.50	62.9588	65.5704	148.4712
		1.00	50.3272	51.1592	76.5564
	Contamination-Normal (eps = 0.05)	0.25	55.2164	95.9604	179.5692
		0.50	46.1244	64.2124	150.1408
		1.00	39.2864	50.5768	77.7036
	t (df = 3)	0.25	24.9748	81.3604	171.8848
		0.50	24.9816	59.4140	147.2580
		1.00	23.4140	47.4508	87.6812
	Laplace (std)	0.25	86.8740	152.2912	149.6004
		0.50	61.2096	72.6232	105.5828
		1.00	46.7076	54.1724	55.9468
0.1	Normal	0.25	103.5560	101.5756	178.0784
		0.50	63.9400	65.3060	147.5964
		1.00	50.3964	51.3020	77.8188
	Contamination-Normal (eps = 0.10)	0.25	38.5976	93.9120	179.4684
		0.50	33.7528	62.3756	151.5264
		1.00	31.5176	49.0216	86.9208
	t (df = 3)	0.25	25.0652	81.9688	160.5184
		0.50	23.3280	59.6376	146.7600
		1.00	22.9008	47.7596	95.2652
	Laplace(std)	0.25	86.6568	151.0972	148.3424
		0.50	60.4788	73.9928	110.9764
		1.00	47.4360	53.7284	56.0240

For small mean shifts, the classical EWMA chart performs well only under the ideal normal model, and its detection performance rapidly deteriorates under contamination and heavy tails. Although the non-adaptive robust EWMA chart achieves improved robustness, it is often slower to detect increases due to conservative fixed limits. The proposed AR-EWMA chart achieves improvements in the detection performance over the non-adaptive robust EWMA chart while maintaining in-control stability. Lastly, all charts detect faster for moderate and large mean shifts as expected. But, the AR-EWMA chart is competitive with the classical EWMA under normality and outperforms the two benchmark schemes under contaminated and heavy-tailed scenarios, demonstrating the capability of the proposed scheme in balancing robustness and detection efficacy across a wide spectrum of operating conditions.

4.3 Effect of Contamination Level

As a demonstration of robustness, Figure 1 also shows the relationship between the in-control average run length ARL0 and the contamination proportion ε.

Figure 1 shows that the classical EWMA chart's ARL_0 rapidly declines even for small contamination levels, which confirms this chart's extreme sensitivity to contamination. Although the ARL_0 of the non-adaptive robust EWMA chart declines more slowly, this chart also fails to maintain a consistent false alarm rate as contamination increases. In contrast, the AR-EWMA chart's almost constant ARL_0 over the entire range of contamination demonstrates the effectiveness of the adaptive scale adjustment in maintaining a consistent false alarm rate.

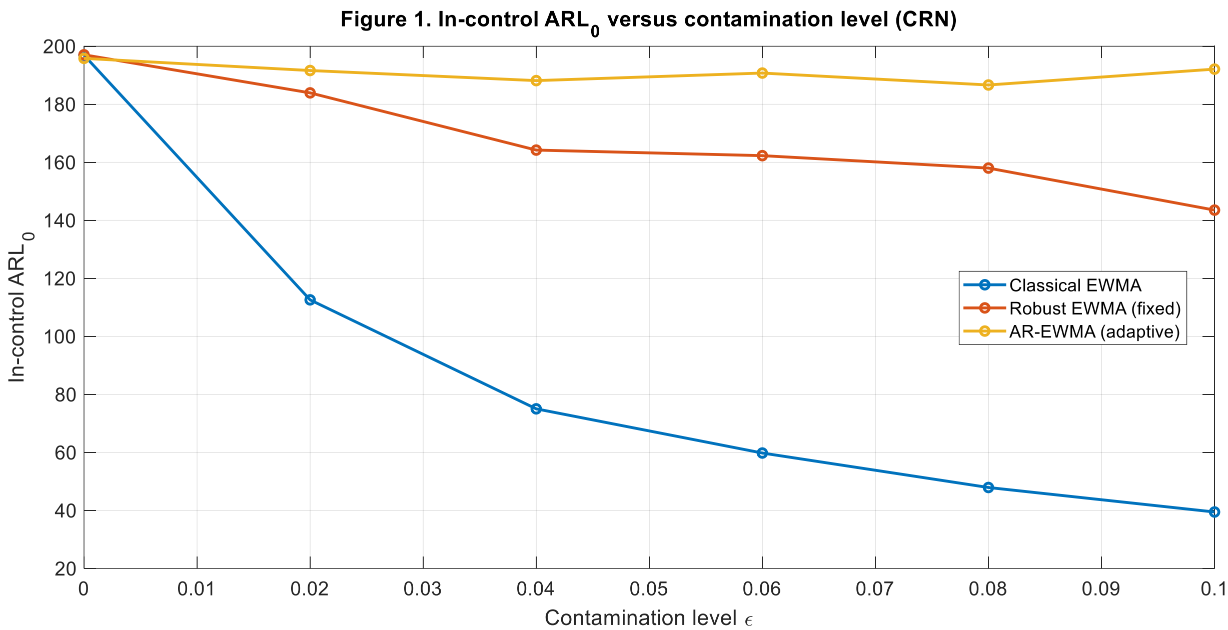

Figure 1. In-control average run length (ARL_0) as a function of the contamination level $\varepsilon$for the classical EWMA, the robust EWMA with fixed limits, and the proposed adaptive robust EWMA (AR-EWMA) chart. All charts are calibrated to achieve $ARL_0 \approx 200$ under the uncontaminated normal model using common random numbers.

4.4 Summary of Results

In summary, the simulation results indicate that the proposed AR-EWMA chart meets the three design objectives satisfactorily: robust stability in false alarms in the case of heavy-tailed or contaminated baseline distributions; competitive out-of-control performance with respect to detecting small and moderate mean shifts; and reduced variability in the run length.

These observations show that merely having robustness is not sufficient for an EWMA to perform effectively in today's data environment; rather, the robustness must be combined with adaptive calibration to ensure stability and efficiency.

5. Conclusions and Future Work

This study develops an Adaptive Robust EWMA (AR-EWMA) control chart designed to monitor shifts in the mean when the in-control process is subject to contamination and heavy-tailed noise. The proposed chart combines two complementary features:

(i) incorporation of bounded-influence updating by way of a robust score function in the EWMA recursion, and

(ii) Adaptive, data-driven control limits based on an online robust scale estimator.

(iii) Together, these two components address the practical need for stable false alarm performance under realistic baseline conditions, where classical EWMA charts often fail to perform reliably.

Based on the simulation results, it is clear that under contamination and heavy-tailed distributions, the classical EWMA chart can exhibit severe false alarm inflation and correspondingly highly unstable run-length behavior. A non-adaptive robust EWMA can achieve better resistance to isolated outliers, but its in-control performance is still vulnerable to deterioration as the contamination level increases. In contrast, the proposed AR-EWMA chart is able to maintain the in-control average run length close to the nominal target under a wide range of contamination levels and heavy-tailed distributions, while producing competitive detection performance for small and moderate mean shifts. Also, the AR-EWMA chart significantly reduces the run-length variability, resulting in more predictable monitoring behavior and more sound decision-making in practice.

From the viewpoint of utilizing statistical process control methodology, the take-home message is that being robust is not enough to achieve a meaningful EWMA chart under contemporary data circumstances. One must also equip the state of robustness with an adaptive calibration to maintain stability of the false alarm rate in the face of evolving effective noise level resulting from contamination or heavy-tailed effects. The AR-EWMA chart provides a ready recipe for doing so, with practically no added computational cost.

Several future research directions can be identified. First, extensions for jointly monitoring the location and scale could be derived by combining the AR-EWMA statistic with an additional robust scale monitoring statistic. Second, the robust methodology can be extended to the multivariate and high-dimensional settings using robust depth or spatial sign transformations. Third, a self-starting version that does not need any Phase I estimation would further enhance the applicability to short-run or non-stationary processes. Fourth, further theoretical results on the asymptotic properties of the adaptive robust EWMA schemes, in particular the stability of its run-length distribution in the presence of contamination, would further justify and support the application of this adaptive robust monitoring scheme.

Overall, the proposed AR-EWMA chart provides a reliable and effective alternative to classical EWMA monitoring in contaminated and heavy-tailed environments, achieving stable false alarm performance and efficient detection of mean shifts.

6. Practical Guidelines and Reproducibility

This section provides implementation guidance for the proposed Adaptive Robust EWMA (AR-EWMA) chart, enabling the reproduction of the presented results.

6.1 Recommended Parameter Settings

Based on the simulation results, the selected parameter values achieve a favorable balance between detection sensitivity and false alarm stability in practical applications:

- Smoothing parameter (λ): Moderate values in the range [0.1, 0.3] are recommended. Sensitivity to small persistent shifts is enhanced by smaller values, while faster detection of moderate shifts is promoted by larger values.
- Robust score function and tuning constant (ψ, c): The Huber score function with $c = 1.345$ provides a good balance between robustness and efficiency for both normal and heavy-tailed distributions. For severe contamination, one may prefer the Tukey bi-square function.
- Rolling window size for robust estimation: Window sizes between 30 and 60 observations produce stable online estimates for location and scale without causing excessive delay.
- Control-limit constant (L): Choose the control-limit constant utilizing the data-driven calibration strategy outlined in Section 2.7 to achieve a desired in-control average run length (e.g., ARL0 ≈ 200).

These guidelines are intended to support routine implementation and may be adjusted to reflect specific process characteristics.

6.2 Computational Considerations

The AR-EWMA chart is computationally light; at each time point, only simple updating of the EWMA statistic and of the robust estimators of location and scale is necessary. As demonstrated by the MATLAB implementation, the proposed methodology can be applied in real-time monitoring or in massive simulation experiments without particular hardware or software requirements.

6.3 Reproducibility Statement

All of the simulation results presented in this paper can be reproduced exactly by employing the MATLAB code provided by the authors. This code implements the AR-EWMA chart and competing EWMA's as well as their calibrations using common random numbers when applicable. Output tables and figures are produced from the Monte Carlo simulations and exported in standard formats for ease of independent replication.

6.4 Final Remark

By combining robustness, adaptivity, and principled calibration, the proposed AR-EWMA chart offers a practical and theoretically grounded solution for detecting mean shifts in contaminated and heavy-tailed processes. The implementation guidelines and reproducible code provided here help the easy adoption of the proposed methodology in both research and industrial settings.

Acknowledgments
The authors are very grateful to the College of Administration and Economics, Salahaddin University, which helped improve this work's quality.

Conflict of Interest
The authors declare that there are no conflicts of interest regarding the publication of this manuscript.

Ethical Approval
Ethical approval was not required for this study as it did not involve human participants, personal data.

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لوحة المتوسط المتحرك الأسي المرجح (EWMA) الحصين التكيفي لمراقبة تغيرات المتوسط في العمليات الملوثة وذات الذيول الثقيلة

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الخلاصة: تُستخدم مخططات المتوسط المتحرك الأسي المرجح (EWMA) على نطاق واسع في مراقبة العمليات الإحصائية، لا سيما للكشف عن التغيرات الصغيرة والمتوسطة في متوسط العملية. إلا أن المخططات الكلاسيكية من هذا النوع تعتمد بشكل كبير على افتراض التوزيع الطبيعي، كما أنها شديدة الحساسية للقيم المتطرفة وحتى لمستويات معتدلة من التلوث، مما قد يؤدي إلى زيادة كبيرة في معدلات الإنذار الكاذب وسلوك غير مستقر لمتوسط طول التشغيل. في هذه الدراسة، نقترح مخطط EWMA قويًا تكيفيًا قادرًا على الحفاظ على استقرار أداء الإنذارات الكاذبة في ظل التوزيعات الملوثة وذات الذيول الثقيلة. يعتمد المخطط المقترح على دمج دالة تأثير قوية ومقيدة ضمن معادلة التحديث، إلى جانب استخدام مقدّر قوي آني للمقياس بهدف ضبط حدود السيطرة بصورة تكيفية. ويهدف هذا التصميم القائم على البيانات إلى تحقيق اتساق تقريبي لمتوسط طول التشغيل في حالة السيطرة عبر مستويات مختلفة من التلوث وأنماط الذيول الثقيلة. وقد أظهرت نتائج المحاكاة أن المخطط المقترح يحقق استقرارًا أفضل في الإنذارات الكاذبة مقارنةً بالمخططات الكلاسيكية والمخططات القوية غير التكيفية، مع احتفاظه بكفاءة كشف تنافسية تحت ظروف غير مثالية. وتشير هذه النتائج إلى أن المخطط التكيفي القوي المقترح يُعد أداة عملية وموثوقة لمراقبة العمليات الصناعية الحديثة في ظل بيانات لا تحقق الافتراضات المثالية.

الكلمات المفتاحية: إحصاءات قوية ؛ التوزيعات الملوثة ؛ العمليات الثقيلة الذيل ؛ التحكم في العملية الإحصائية ؛ استقرار الإنذار الكاذب.