



Quality Control Charts for Multivariate EWMA Daubechies Discrete Wavelet Transformation Coefficients

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Abstract

Multivariate exponential weighted moving averages (MEWMA) are used to control several qualitative properties together of production processes. This article proposes the creation of new charts to control and monitor multivariate qualitative property exponential weighted moving averages, as well as variance through wavelet analysis based on the discrete wavelet transform (Daubechies). Wavelet analysis breaks down multivariate data into approximation and detail coefficients, which are used to construct the Exponential Weighted Moving Averages for approximation coefficients (MAEWMA) chart (to control and monitor the average) and the Exponential Weighted Moving Averages for detail coefficients (MDEWMA) chart (to control and monitor the variance). The proposed charts were more efficient than the conventional chart and more sensitive to slight changes, that can occur in the production processes at several values of the tuning parameter and for different sample sizes and numbers of variables through simulation studies and real data.

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1. Introduction

Quality control (QC) is essential for monitoring and sustaining process stability across diverse industries, guaranteeing that goods continually adhere to quality requirements. Conventional control charts, such as Shewhart and Exponentially Weighted Moving Average (EWMA) charts, are frequently employed to identify alterations in process parameters (Mallat, 1999). Many contemporary processes entail sophisticated, high-dimensional data, defying effective observation with conventional univariate techniques. To classify this challenge, Multivariate EWMA (MEWMA) charts were created to control many correlated variables concurrently, developing the identification of subtle process varies that may not be tangible proof when assessing individual parameters in isolation (Ali et al., 2024). Lately, the Daubechies Discrete Wavelet Transformation (DWT) has emerged as a prevalent and potent tool for data assessment, particularly in signal processing and time-series analysis (Daubechies, 1992). DWT efficiently decomposes complicated, non-stationary signals into separate frequency components while maintaining temporal information, deriving it particularly effective for identifying process shifts and abnormalities in quality control (Ali & Ali, 2019). Daubechies wavelets, a class of orthogonal wavelets with efficient support, are particularly beneficial for data decomposition and analysis because they effectively capture both slow and sudden signal changes (Percival & Walden, 2000). Integrating Multivariate EWMA with Daubechies DWT coefficients is a practical approach for quality control. It facilitates the concurrent monitoring of many variables and detects nuanced changes across both temporal and frequency domains (Abramovich et al., 2000).

This combination improves the capacity to identify subtle, incremental process shifts that conventional control charts may overlook. This methodology is especially beneficial in sectors characterized by intricate activities, such as manufacturing, chemical processing, and telecommunications, where signals or measurements frequently display temporal and geographical dependencies (Ali et al., 2023).

This article shows novel charts elevated to manage and oversee multivariate quality attributes by applying Exponentially Weighted Moving Averages (EWMA) parallel to wavelet analysis, particularly the Daubechies Discrete Wavelet Transform (DWT). Wavelet analysis putrefies multivariate data into approximation and detail coefficients. These are subsequently used to run two separate charts: the MAEWMA chart for controlling the average through approximation coefficients and the MDEWMA chart for tracking variation via detail coefficients. The research shows that the proposed charts exhibit greater efficiency and sensitivity to minor variations in creating processes than traditional ways, as evidenced by simulations and empirical data across various parameter values, sample sizes, and variables.

2. Quality Control Charts

Quality control charts, or statistical process control (SPC) charts, are vital instruments employed in manufacturing to monitor and run processes within time (Ali & Haydier, 2016). They are well-focused in identifying variations or anomalies in a process that may lead to errors or deviations from what we wait for (Ali & Saleh, 2022). These charts generally provide plotted data points in conjunction with control limits that indicate the expected variation within the process (Montgomery, 2019). Data points that go up certain thresholds or show non-random patterns suggest that the process may be uncontrolled, warranting additional scrutiny and remedial measures. Quality control charts guarantee consistent product quality and enhance processes (Sullivan & Woodall, 1996).

3. MEWMA Chart

The Multivariate Exponentially Weighted Moving Average (MEWMA) chart is a statistical process control tool that tracks the mean vector of multiple interrelated quality indicators in a multivariate process over time (Wierda, 1994; Ali et al. 2017). It employs an exponentially weighted average of prior notes, attributing greater greatness to more recent data points and developing sensitivity to slight variations in the process mean (Mason & Young, 2002). Mathematically, the MEWMA statistic Z_t at time t is calculated using the recursive formula:

$$Z_t = \lambda X_t + (1 - \lambda)Z_{t-1} \tag{1}$$

Where X_t is the noted data vector at the time t , λ is the smoothing parameter ($0 < \lambda \leq 1$), and Z_0 is typically summarized to the in-control mean vector. The upper and lower control limits of the MEWMA chart:

$$\begin{matrix} \text{UCL} \\ \text{LCL} \end{matrix} = \bar{Z} \pm 3 \times \sqrt{\lambda/(2 - \lambda)} \times |\text{CZ}| \tag{2}$$

Where \bar{Z} is the average of the Z-score values and CZ is a covariance matrix. The MEWMA chart simplifies the identification of deviations from expected performance by comparing Z_t to control limits derived from the covariance structure of the process (Alt, F.B. 1985). This enables timely corrective actions to maintain steady quality standards across numerous industries (Montgomery, 2020).

4. Wavelet Analysis

Wavelet analysis is a mathematical technique used for assessing and processing signals, offering localized representations in both time and frequency domains (Daubechies, 1992; Zhang, & Wang, 2022). It employs wavelets—compact, oscillatory functions that can be scaled and shifted—to identify transient features inside signals. A wavelet function $\psi(t)$ is scaled and shifted as follows:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \tag{3}$$

where $a > 0$ is the scale parameter controlling dilation or compression, and b is the translation parameter determining the shift. The continuous wavelet transforms (CWT) of a signal $f(t)$ is defined by:

$$W_f(a, b) = \int_{-\infty}^{\infty} f(t)\psi_{a,b}^*(t)dt \tag{4}$$

Where * denotes the complicated conjugate of the wavelet function. This change simplifies the identification of transient features, making it beneficial for noise decrease, signal compression, and feature extraction applications. Unlike standard Fourier analysis, which provides global frequency information, wavelet analysis facilitates multi-resolution analysis, allowing a comprehensive examination of signals across several scales (Jalal et al., 2024). The versatility of wavelet analysis renders it pertinent across several domains, such as engineering, finance, and medicinal sciences (Percival & Walden, 2000).

5. Daubechies Wavelet

The Daubechies wavelet, developed by Ingrid Daubechies, is a renowned family of wavelets well known for its orthogonality and compact support (Ali et al., 2024). These wavelets are exceptionally skilled in applications such as picture compression, noise decrease, and feature extraction (Daubechies, 1992). A defining characteristic is compact support, indicating they are non-zero only within a restricted range, rendering them optimal for thorough signal analysis (Ali & Albarwari, 2023). Mathematically, the scaling function $\Phi(t)$ and wavelet function $\psi(t)$ of the Daubechies wavelet satisfy the following two-scale relation:

$$\phi(t) = \sqrt{2} \sum_k h_k \phi(2t - k) \quad (5)$$

$$\psi(t) = \sqrt{2} \sum_k g_k \phi(2t - k) \quad (6)$$

Where h_k and g_k filter coefficients derived to ensure orthogonality and compact support, their orthogonal characteristics guarantee that signals can be decomposed and rebuilt without redundancy. Daubechies wavelets are scalable, enabling multi-resolution analysis to examine signals at many scales (Percival & Walden, 2000). They excel in detecting sudden changes or localized characteristics, rendering them fit for analyzing non-stationary data. These wavelets are used in data compression, medical signal processing, and time-series analysis (Daubechies, 1992).

6. Discrete Wavelet Transformation

The Discrete Wavelet Transform (DWT) is a mathematical technique that decomposes a signal into its major components along with many frequency levels, providing visions into both time and frequency domains (Ali and Awaz, 2017). Mathematically, the DWT of a signal $x(t)$ is expressed as:

$$W(j, k) = \sum_j^n x(n) \psi_{j,k}(n) \quad (7)$$

where $\psi_{j,k}(n)$ are the discrete wavelet basis functions obtained by scaling and translating the mother wavelet $\psi(t)$:

$$\psi_{j,k}(t) = 2^{-j/k} \psi(2^{-j}t - k) \quad (8)$$

with j and k representing the scale and translation indices, respectively. Vice versa to the steady wavelet transformation, the DWT uses discrete values for scaling and translating, resulting in devoted processing efficiency (Abramovich et al. 2000). Using filter banks, DWT separates a signal $x(n)$ into approximation coefficients $A(n)$ (low-frequency components) and detail coefficients $D(n)$ (high-frequency components) through the following relations:

$$A(n) = \sum_k^{L-1} h(k) x(n - k) \quad (9)$$

$$D(n) = \sum_k^{L-1} g(k) x(n - k) \quad (10)$$

where L is the number moment vanishing, and $h(k)$ and $g(k)$ are the low-pass and high-pass filter coefficients, respectively. This process is recursive, enabling multi-resolution analysis where the approximation coefficients are further decomposed into new approximation and detail coefficients. DWT is elastic and used in signal processing for data compression, noise decrease, and feature taking out, having applications in view and audio processing, as well as time-series analysis. Its capacity to provide localized time-frequency information renders it especially advantageous for analyzing non-stationary signals (Mallat, S., 1999).

7. Proposed Charts

To create new charts based on the discrete wavelet transform of the Daubechies wavelet, specifically to create a chart of approximation (for average) and detail (for variance) coefficients for the Daubechies wavelet for the MEWMA. Suppose that X is a data matrix, where the columns represent several variables (p), and the rows represent the observations (n) for even number n. Perform a discrete wavelet transform for Daubechies wavelet of order (N = 1, 2, ..., 45) for each variable of the data matrix to produce (n/2+N -1) of the approximate and detailed coefficients (two partitions) as in the following formulas:

$$A(s) \equiv V_{j,i} = \sum_{l=0}^{L-1} g_l V_{j-1,2i+1-l \text{ mod } N_{j-1}} \tag{11}$$

$$D(s) \equiv W_{j,i} = \sum_{l=0}^{L-1} h_l V_{j-1,2i+1-l \text{ mod } N_{j-1}} \tag{12}$$

Define $V_0 = x$ and x is the observations vector of length (n) and set $j = 1$ (the level) input to the j^{th} stage of the pyramid algorithm is V_{j-1} (is full-band), and related to frequencies $[0, 1/2^j]$ in x. Half-band filters for $i = 0, 1, \dots, N_j-1$. Formula 1 represents the approximation coefficients (first part), scale or father function A(s) and $s = 1, 2, \dots, p$ with (n/2+N-1) coefficients, which are proportional to the qualitative characteristics of observations summation at each level L. In contrast, formula 2 represents (second part) the detail coefficients D(s), the mother or wavelet function with (n/2+N-1) coefficients, which are proportional to the differences (variance) of the observations of the qualitative characteristic. Repeat the calculation of the discrete wavelet transform for Daubechies wavelet of order (1, 2, ..., 45) for each column of the data matrix X. The following matrices of multivariate approximation and detail coefficients are obtained:

$$A = [A(1) A(2) \dots A(p)] \tag{13}$$

$$D = [D(1) D(2) \dots D(p)] \tag{14}$$

The proposed charts are based on Daubechies wavelet analysis, which involves creating two charts, the first for controlling the MEWMA Daubechies approximation coefficients (MAEWMA) and the second for controlling the MEWMA Daubechies detail coefficients (MDEWMA). Calculating mean vector for approximate and detailing coefficients (A and D):

$$\bar{A} = [\bar{A}(1) \bar{A}(2) \dots \bar{A}(p)] \tag{15}$$

$$\bar{D} = [\bar{D}(1) \bar{D}(2) \dots \bar{D}(p)] \tag{16}$$

\bar{A} represents the approximation coefficients mean vector and \bar{D} represents the detail coefficients mean vector. Estimation of variance and covariance matrix for approximation and detail coefficients (CA and CD):

$$CA = \begin{bmatrix} V(A(1,1)) & COV(A(1,2)) & \dots & COV(A(1,p)) \\ COV(A(2,1)) & V(A(2,2)) & \dots & COV(A(2,p)) \\ \vdots & \vdots & \ddots & \vdots \\ COV(A(p,1)) & COV(A(p,2)) & \dots & V(A(p,p)) \end{bmatrix} \tag{17}$$

$$CD = \begin{bmatrix} V(D(1,1)) & COV(D(1,2)) & \dots & COV(D(1,p)) \\ COV(D(2,1)) & V(D(2,2)) & \dots & COV(D(2,p)) \\ \vdots & \vdots & \ddots & \vdots \\ COV(D(p,1)) & COV(D(p,2)) & \dots & V(D(p,p)) \end{bmatrix} \tag{18}$$

Calculate the MAEWMA and MDEWMA statistic for the approximation and detail coefficients respectively, which are the points drawn on the proposed charts at tune parameter $0 < \lambda < 1$. Initialize the MAEWMA and MDEWMA statistics as follows:

$$ZA(1) = \lambda \times A(1, :) \tag{19}$$

$$ZD(1) = \lambda \times D(1, :) \tag{20}$$

Where $A(1, :)$ and $D(1, :)$ represent the first row for the approximate and detail coefficients matrix respectively. For $i = 2, 3, \dots, n = \text{Length}(A(1))$. $ZA(i)$ and $ZD(i)$ are:

$$ZA(i) = \lambda \times A(i, :) + (1 - \lambda) \times ZA(i - 1, :) \tag{21}$$

$$ZD(i) = \lambda \times D(i, :) + (1 - \lambda) \times ZD(i - 1, :) \tag{22}$$

The upper and lower control limits of the MAEWMA and MDEWMA charts:

$$\begin{matrix} \text{UCLA} \\ \text{LCLA} \end{matrix} = \text{mean}(ZA) \pm 3 \times \sqrt{\lambda/(2 - \lambda)} \times |CA| \tag{23}$$

$$\begin{matrix} \text{UCLD} \\ \text{LCLD} \end{matrix} = \text{mean}(ZD) \pm 3 \times \sqrt{\lambda/(2 - \lambda)} \times |CD| \tag{24}$$

CA and CD were obtained from equations 17 and 18 respectively.

8. Simulation Study

To clarify the process of converting the original observations to the approximation and detail coefficients only and not to build the proposed qualitative control chart, (10) random observations were generated from a multivariate $N(10, I)$, and the discrete wavelet transform for the Db6 wavelet (using MATLAB) was used at the first level to obtain three values for the approximation coefficients and three values for the detail coefficients, as in Figure 1.

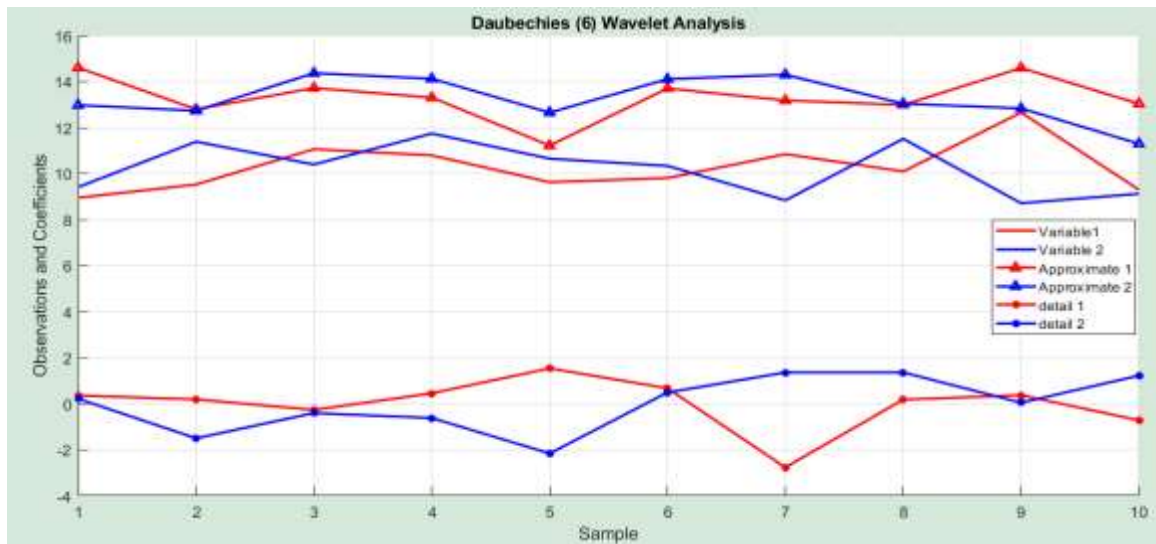


Figure 1. Daubechies (6) Wavelet Analysis

Figure 1 The analysis illustrates the decomposition of multivariate data into approximation and detail coefficients utilizing the Daubechies (6) wavelet. This method facilitates the efficient oversight of qualitative data using the suggested MEWMA-based charts: MAEWMA for means and MDEWMA for variances. The decomposition distinctly differentiates between trends and noise, as illustrated in the picture. Approximation coefficients encapsulate smoothing patterns, which are beneficial for identifying shifts in means, whereas detail coefficients highlight variability and anomalies, aiding in the observation of changes in variance.

The proposed charts and the points drawn on them for the first experiment simulation, (50) random observations were generated from a multivariate $N(\mu, I)$, for $\mu = [10; 10]$ and the discrete wavelet transform for the Db6 wavelet (using MATLAB). Figure 2 illustrates the MEWMA (Multivariate Exponentially Weighted Moving Average) chart utilized to monitor two variables over 50 samples. The chart controls the weighted moving averages of the variables, assuming their multivariate correlation structure. The blue and red lines show the MEWMA statistics for the two variables. Vice versa, the dashed lines show the upper control limit (UCL) and lower control limit (LCL), derived from the statistical characteristics of the data and the tuning parameter λ . The graphic shows that both variables steadily maintain within

the control boundaries, implying process stability. The oscillations indicate fluctuations in the data, with no indication of substantial changes or trends going up the control levels.

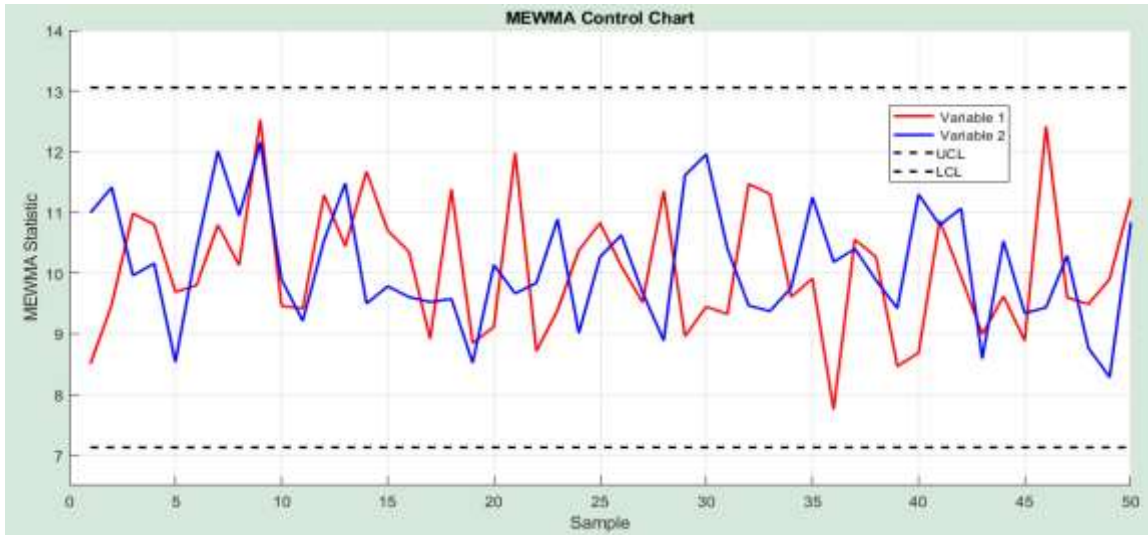


Figure 2. MEWMA Chart for first experiment simulation

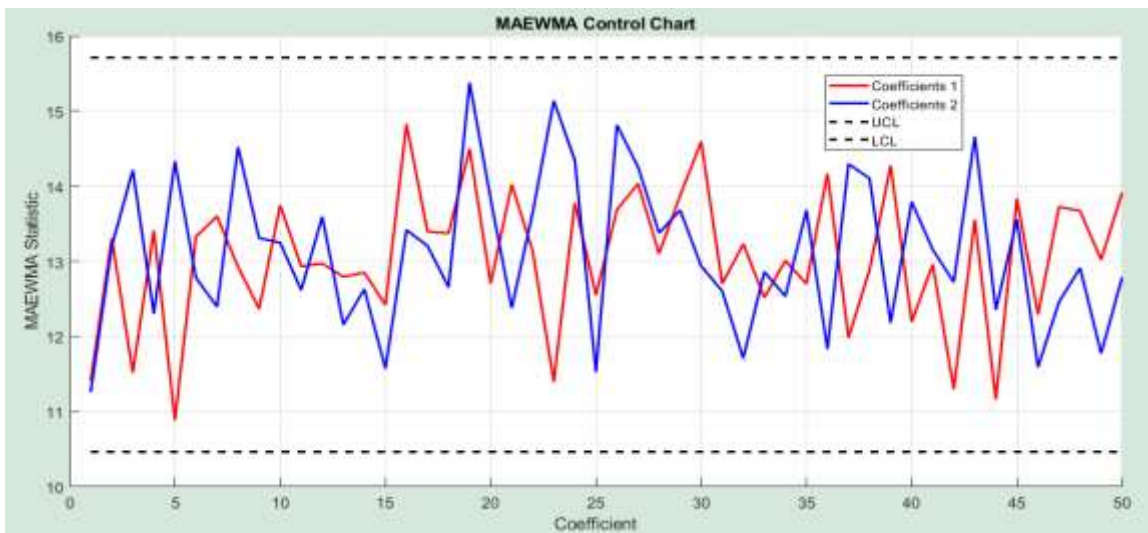


Figure 3. MAEWMA Chart for first experiment simulation

Figure 3 depicts the MAEWMA control curve for the approximation coefficients derived from the initial experiment simulation. The chart displays two independent lines—red and blue—illustrating the calculated statistics for two different sets of coefficients (Coefficients 1 and 2) over several rounds. The dashed lines on the graph represent the upper control limit (UCL) and lower control limit (LCL), which act as thresholds for finding anticipated irregularities in the monitored process. The graphic indicates that the data points primarily stay inside the control limits, with tiny variations. The results illustrate that the process is stable, exhibiting no notable aberrations that require further examination.

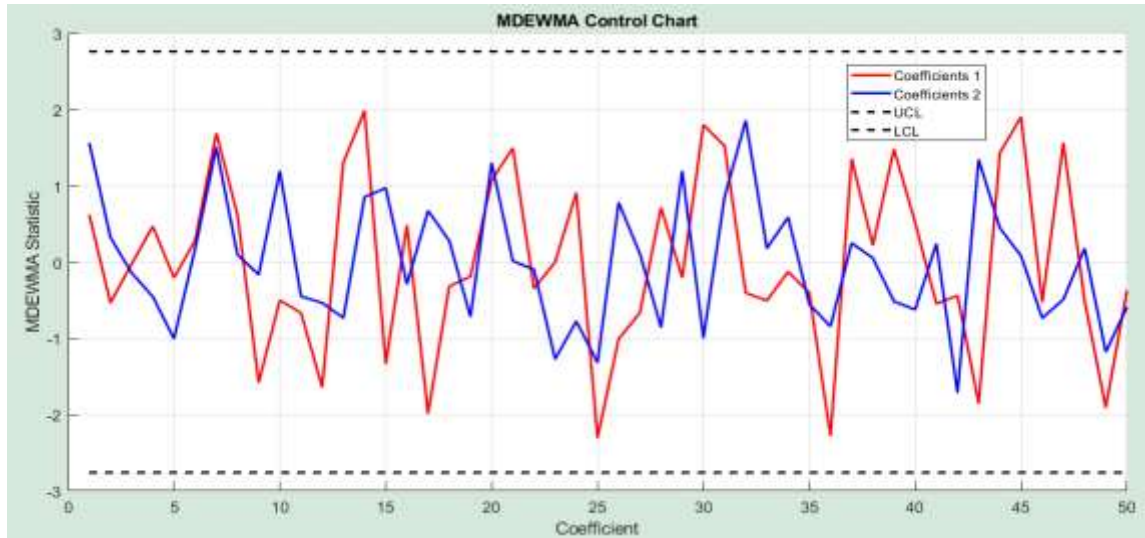


Figure 4. MDEWMA Chart for First Experiment Simulation

The MDEWMA control chart (Figure 4) features two lines (red and blue), demonstrating the MDEWMA statistics for two coefficient sets ("Coefficients 1" and "Coefficients 2"), which control the departure of noted variables from expected performance over time. The black dashed horizontal lines denote the Upper Control Limit (UCL) and Lower Control Limit (LCL), establishing the permissible range for process behaviour. At the same time, any points above these limits indicate a potential out-of-control process. The x-axis represents the time index or sequence of observations (1 to 50), whilst the y-axis denotes the deviation of the multivariate process from the goal mean. Observations indicate that the MDEWMA statistics oscillate within the control boundaries, no points exceed the limits, signifying that the process is under control. The C values represent the disparities between UCL and LCL.

Table 1. Results of Chats for First Experiment Simulation

Chart	Mean	Determinant	UCL	LCL	C
MEWMA	10.0916	1.0792	13.0560	7.1272	6.3985
MAEWMA	13.0882	0.8449	15.7112	10.4652	4.4324
MDEWMA	0.0005	0.9357	2.7607	-2.7597	5.1653

Table 1 displays the performance metrics for the three categories of monitoring charts—MEWMA, MAEWMA, and MDEWMA—derived from the initial experimental simulation. The metrics comprise the mean, determinant of the covariance matrix, upper control limit (UCL), lower control limit (LCL), and a parameter C, which signifies a measure of total sensitivity. The MEWMA chart demonstrates strong control efficacy with modest responsiveness to process deviations. Consequently, a sensitivity of 4.4324 signifies The MAEWMA chart, which tracks the average (approximation coefficients), exhibits heightened sensitivity to minor fluctuations in the mean, with narrower boundaries and enhanced monitoring efficacy. The MDEWMA chart tracks variability (detail coefficients) and demonstrates enhanced efficacy in identifying variance-related process alterations relative to the conventional MEWMA chart.

The simulation experiments were repeated (1000 for ($p = 2, 3,$ and $4,$ Average = $[10 \ 10]$, Variance and Covariance equal to identity matrix) with different values of λ (By experimenting with different values of λ and analyzing the resulting control chart, you can choose the best λ that meets your needs) using Daubechies 26. The results are in Tables (2-4):

Table 2. Results Average of Chats for Simulation ($p = 2$)

Chart	λ	n	Mean	Determinant	UCL	LCL	C
MEWMA			9.9492	0.5298	11.7123	8.1860	1.9513
MAEWMA	0.80	50	12.9025	0.4505	14.5156	11.2894	1.5658
MDEWMA			-0.0001	0.4018	1.5174	-1.5172	1.3337

MEWMA		9.9745	0.4888	11.6772	8.2719	1.7029
MAEWMA	100	12.9039	0.4428	14.5061	11.3018	1.5178
MDEWMA		-0.0001	0.4101	1.5329	-1.5331	1.3763
MEWMA		9.9639	0.5895	11.9223	8.0055	2.4103
MAEWMA	50	12.9216	0.4654	14.6465	11.1967	1.7348
MDEWMA		-0.0001	0.4968	1.7779	-1.7777	1.9265
MEWMA	0.85	9.9819	0.5695	11.9173	8.0465	2.2549
MAEWMA	100	12.9226	0.4566	14.6340	11.2112	1.6785
MDEWMA		-0.0001	0.5078	1.7973	-1.7974	1.9924
MEWMA		9.9770	0.6774	12.1860	7.7679	3.1232
MAEWMA	50	12.9385	0.5062	14.8301	11.0470	2.0731
MDEWMA		0.0000	0.6113	2.0762	-2.0761	2.7624
MEWMA	0.90	9.9885	0.6754	12.2060	7.7709	3.0630
MAEWMA	100	12.9392	0.4956	14.8135	11.0649	2.0020
MDEWMA		-0.0000	0.6259	2.1006	-2.1007	2.8646
MEWMA		9.9887	0.8015	12.5156	7.4617	4.2263
MAEWMA	50	12.9537	0.5807	15.0840	10.8234	2.6802
MDEWMA		-0.0000	0.7497	2.4188	-2.4187	3.9398
MEWMA	0.95	9.9943	0.8126	12.5524	7.4363	4.2508
MAEWMA	100	12.9540	0.5673	15.0616	10.8465	2.5828
MDEWMA		-0.0000	0.7691	2.4495	-2.4495	4.0991

Table 2 displays the findings of simulation research assessing the efficacy of the proposed MEWMA, MAEWMA, and MDEWMA charts for multivariate qualitative attributes with two variables ($p = 2$). The outcomes are presented for various tuning parameter values ($\lambda=0.80, 0.85, 0.90, 0.95$) and sample sizes ($n = 50, n = 100$). The MEWMA chart regularly yields lower mean values than both MAEWMA and MDEWMA. The MAEWMA graphic demonstrates elevated mean values for all tuning parameters, signifying an enhanced capacity to detect changes in the mean of approximation coefficients. The mean values of the MDEWMA chart are centered at 0, as anticipated for variance monitoring. The MAEWMA chart exhibits lower determinant values compared to MEWMA, signifying enhanced sensitivity to multivariate alterations in approximation coefficients. MDEWMA has the same patterns for detail coefficients, with determinants experiencing a modest increase as λ rises.

The upper and lower control limits (UCL and LCL) for MAEWMA and MDEWMA are consistently broader than those of MEWMA, particularly at elevated λ , indicating the enhanced sensitivity of wavelet-based charts to variations in both mean and variance. The C values, representing the disparities between UCL and LCL, underscore the enhanced sensitivity of the wavelet-based charts. Both MAEWMA and MDEWMA exhibit superior control over fluctuations compared to MEWMA, with reduced C-values as λ grows.

Table 3. Results Average of Chats for Simulation ($p = 3$)

Chart	λ	n	Mean	Determinant	UCL	LCL	C
MEWMA			9.9489	0.3718	11.4175	8.4802	1.1659
MAEWMA		50	12.9021	0.2740	14.1472	11.6569	0.7648
MDEWMA	0.80		0.0001	0.2407	1.1615	-1.1614	0.6376
MEWMA			9.9750	0.3359	11.3823	8.5677	0.9787
MAEWMA		100	12.9037	0.2692	14.1388	11.6685	0.7436
MDEWMA			-0.0001	0.2508	1.1853	-1.1855	0.6797
MEWMA			9.9636	0.4423	11.6505	8.2766	1.5917
MAEWMA		50	12.9212	0.2971	14.2856	11.5568	0.9101
MDEWMA	0.85		0.0000	0.3319	1.4377	-1.4376	1.0846
MEWMA			9.9824	0.4240	11.6475	8.3173	1.4608
MAEWMA		100	12.9223	0.2914	14.2742	11.5705	0.8844
MDEWMA			-0.0001	0.3467	1.4688	-1.4689	1.1609

MEWMA			9.9767	0.5477	11.9520	8.0013	2.3060
MAEWMA		50	12.9381	0.3410	14.4756	11.4006	1.1782
MDEWMA			0.0000	0.4543	1.7708	-1.7707	1.8235
MEWMA	0.90		9.9889	0.5481	11.9811	7.9968	2.2577
MAEWMA		100	12.9390	0.3336	14.4592	11.4187	1.1433
MDEWMA			-0.0000	0.4758	1.8113	-1.8114	1.9604
MEWMA			9.9884	0.7055	12.3464	7.6303	3.5442
MAEWMA		50	12.9533	0.4200	14.7476	11.1590	1.6924
MDEWMA			0.0000	0.6181	2.1730	-2.1730	3.0395
MEWMA	0.95		9.9948	0.7234	12.4018	7.5878	3.5987
MAEWMA		100	12.9538	0.4091	14.7228	11.1848	1.6367
MDEWMA			-0.0000	0.6493	2.2258	-2.2259	3.2838

Table 3 delineates the performance of MEWMA, MAEWMA, and MDEWMA charts over various tuning parameter λ values and sample sizes ($n = 50$ and $n = 100$). Essential metrics encompass the mean, determinant of the covariance matrix, upper control limit (UCL), lower control limit (LCL), and sensitivity measure (C). The MEWMA chart exhibits a stable mean (~9.95–9.99) across varying λ values and sample sizes, whereas the determinant escalates with rising λ , indicating enhanced variability in the covariance structure. The control limits expand considerably with increased λ , signifying diminished sensitivity to minor process variations, but sensitivity C rises but remains inferior to that of MDEWMA, demonstrating moderate adaptability.

The MAEWMA chart exhibits a mean of approximately 12.90 across all situations, accompanied by marginally narrower control limits compared to MEWMA. The determinant is constantly inferior to MEWMA, underscoring the chart's efficacy in diminishing covariance-related variability, sensitivity C is markedly elevated, rendering it more reactive to alterations in the process mean.

The MDEWMA chart emphasizes variance tracking rather than mean, with a mean around 0 and a determinant above that of MAEWMA, highlighting its emphasis on variability capture. Sensitivity C exhibits the greatest values across the charts, particularly at larger λ , highlighting its exceptional efficacy in identifying changes in process variance.

Table 4 presents the average performance metrics for the MEWMA, MAEWMA, and MDEWMA charts at several values of the tuning parameter λ and sample sizes ($n = 50$ and $n = 100$). The principal metrics comprise the mean, determinant of the covariance matrix, UCL, LCL, and a sensitivity measure referred to as C. As increases from 0.80 to 0.95, the UCL and LCL of the MEWMA chart expand, signifying enhanced sensitivity to process changes, with the sensitivity measure C rising from 0.6572 to 2.9823. This indicates a more robust reaction to process variations over time, and with increased sample numbers ($n = 100$), the UCL and LCL exhibit minor movements while maintaining consistency, so illustrating the chart's stability.

The MAEWMA chart, which emphasizes the monitoring of averages, exhibits a consistent mean of approximately 12.90, but the UCL and LCL values progressively rise with increasing λ . The sensitivity measure C likewise rises with increased λ , albeit it is lesser than that of the MEWMA chart, signifying reduced volatility. The MDEWMA graphic, intended to track variance (detail coefficients), exhibits a mean approximating 0 throughout all simulations, while the UCL and LCL expand as λ grows, indicating a more pronounced reaction to process variance. Sensitivity, denoted as C, likewise escalates with λ , indicating an enhanced capacity to discern variations in variance, with values spanning from 0.2907 to 2.5005.

Table 4. Results Average of Chats for Simulation ($p = 4$)

Chart	λ	n	Mean	Determinant	UCL	LCL	C
MEWMA			9.9496	0.2505	11.1472	8.7520	0.6572
MAEWMA		50	12.9030	0.1553	13.8305	11.9754	0.3370
MDEWMA			0.0000	0.1387	0.8700	-0.8699	0.2907
MEWMA		100	9.9746	0.2270	11.1284	8.8209	0.5483

MAEWMA			12.9039	0.1493	13.8127	11.9950	0.3188
MDEWMA			0.0000	0.1473	0.8949	-0.8949	0.3199
MEWMA			9.9643	0.3209	11.3918	8.5368	1.0029
MAEWMA		50	12.9221	0.1789	13.9698	11.8743	0.4393
MDEWMA			0.0000	0.2135	1.1376	-1.1376	0.5824
MEWMA	0.85		9.9820	0.3110	11.4042	8.5598	0.9250
MAEWMA		100	12.9226	0.1713	13.9462	11.8990	0.4146
MDEWMA			0.0000	0.2274	1.1723	-1.1722	0.6443
MEWMA			9.9774	0.4299	11.7162	8.2386	1.6355
MAEWMA		50	12.9390	0.2180	14.1548	11.7232	0.6225
MDEWMA			0.0000	0.3247	1.4776	-1.4775	1.1469
MEWMA	0.90		9.9885	0.4385	11.7657	8.2114	1.6283
MAEWMA		100	12.9392	0.2078	14.1230	11.7554	0.5858
MDEWMA			0.0000	0.3473	1.5256	-1.5256	1.2769
MEWMA			9.9891	0.6047	12.1580	7.8201	2.8676
MAEWMA		50	12.9542	0.2882	14.4235	11.4848	0.9958
MDEWMA			0.0000	0.4897	1.9093	-1.9093	2.2293
MEWMA	0.95		9.9944	0.6349	12.2436	7.7451	2.9823
MAEWMA		100	12.9541	0.2733	14.3795	11.5287	0.9333
MDEWMA			0.0000	0.5262	1.9759	-1.9759	2.5005

9. Real Data

The source of this real data is Erbil Steel company which is construction commencing in 2006 in northern Iraq. Erbil Steel Company started integrated steel production in December 2007. The facility which produces its energy through its 32 MW [powerhouse has an annual steel production capacity of 240,000 Tons. This data has been concluding 100 observations on a Nominal diameter of 12 mm which is composed of 2 different variables: Yield point and Tensile strength.

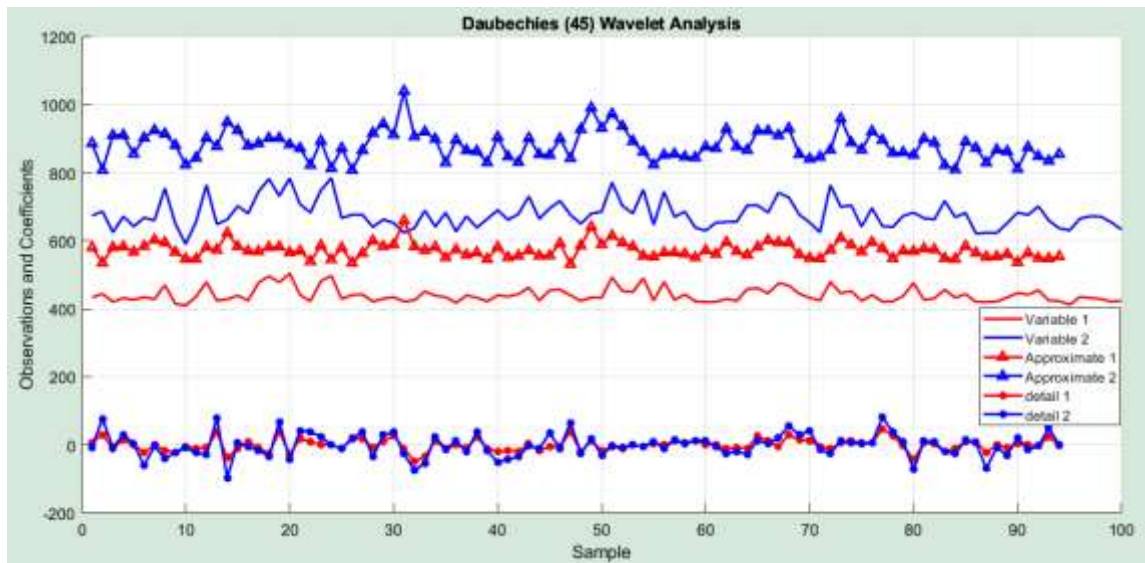


Figure 5. Daubechies (45) Wavelet Analysis for Real Data

The chart depicts the multivariate monitoring procedure with wavelet decomposition and control charts. The upper two lines (blue and red) indicate Variables 1 and 2, indicating initial data observations. Triangular markers below show approximation coefficients obtained from Formula (10), which reflect the average behaviour of the data, and circular markers indicate detail coefficients produced from Formula (11), emphasizing fluctuations or deviations from the average. The data is hierarchically split into approximation and detail coefficients for further analysis using Daubechies

(45) wavelets. These coefficients provide two control charts: the MAEWMA chart, which observes means, and the MDEWMA chart, which controls variance. The notification indicates that although the variables exhibit notable oscillations, the approximation coefficients mitigate these variations, while the detail coefficients capture finer-scale noise, offering insights into general behaviour and localized anomalies.

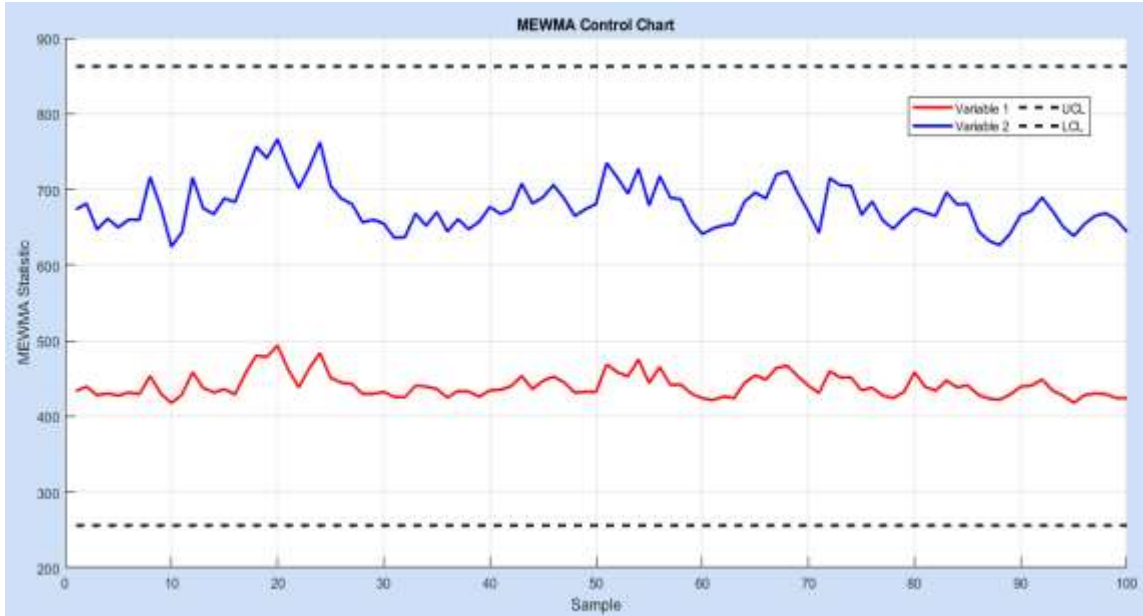


Figure 6. MEWMA Chart for Real Data

Figure 6 presents a Multivariate Exponentially Weighted Moving Average (MEWMA) control chart, illustrating the monitoring of two variables (Variable 1 in red and Variable 2 in blue) over 100 samples. The chart includes dashed black lines representing the Upper Control Limit (UCL) and Lower Control Limit (LCL), which define the acceptable range for process stability. Both variables remain within these control limits throughout the samples, indicating no significant shifts or abnormalities. While variations in the MEWMA statistics are observed, the process appears stable and under control based on the data.

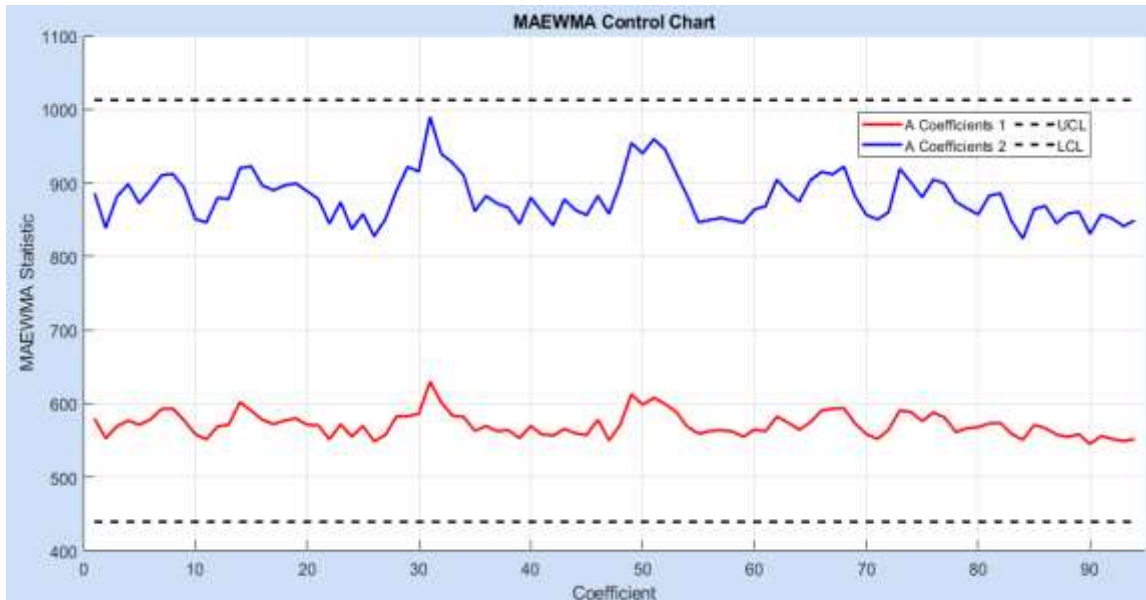


Figure 7. MAEWMA Chart for Real Data

Figure 7 illustrates a Multivariate Approximate Exponentially Weighted Moving Average (MAEWMA) control chart that monitors two coefficients (Coefficient 1 in red and Coefficient 2 in blue) across 100 observations. The chart shows control limits (UCL and LCL) represented by dashed black lines, which define the boundaries for acceptable process performance. Both coefficients remain within these control limits, indicating a stable process without significant deviations or out-of-control conditions. While the blue line shows larger fluctuations compared to the red line, the process maintains overall stability throughout the dataset.

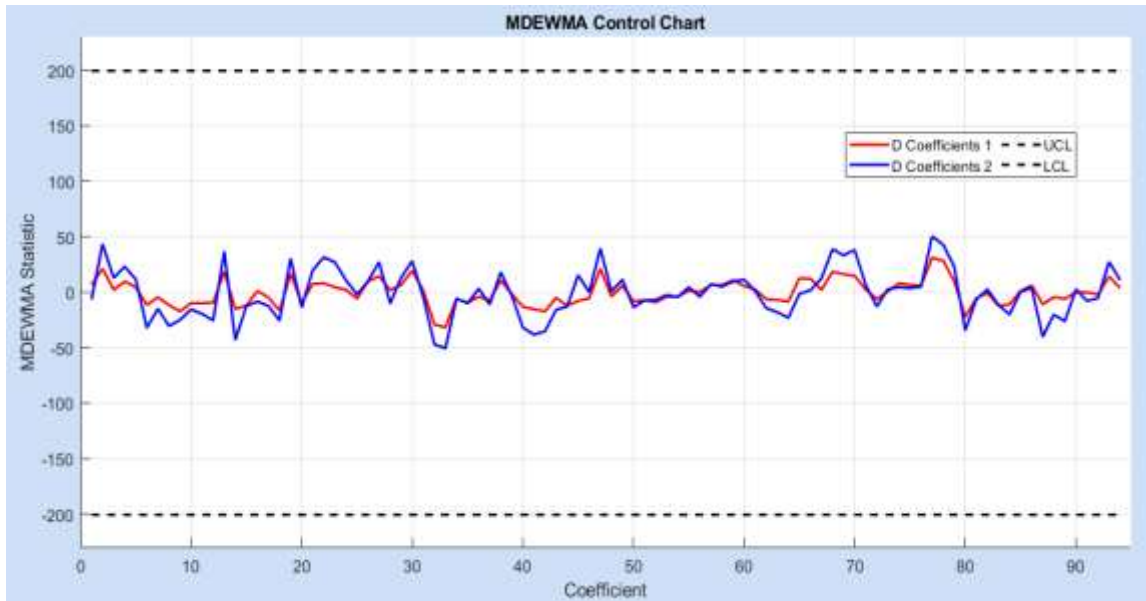


Figure 8. MDEWMA Chart for Real Data

Figure 8 presents a Multivariate Detail Exponentially Weighted Moving Average (MDEWMA) control chart monitoring two coefficients (Coefficient 1 in red and Coefficient 2 in blue) over 100 observations. The control limits, represented by dashed black lines, indicate the range of acceptable variation for the process. The MDEWMA statistics for both coefficients remain well within the upper and lower control limits, showing a stable process with no significant deviations or out-of-control points. The red and blue lines are closely aligned, suggesting minimal variation between the two coefficients throughout the monitored period.

Table 5. Results of Chats for Real Data

Chart	Mean	Determinant	UCL	LCL	C
MEWMA	559.2609	23854	862.5890	255.9327	1.4471
MAEWMA	726.0635	21304	1012.700	439.4033	1.2214
MDEWMA	-0.7283	10360	199.1674	-200.6240	0.4142

Table 5 summarizes the results of applying three control chart methods—MEWMA, MAEWMA, and MDEWMA—to real data, detailing their statistical metrics. MEWMA shows a mean of 559.2609, a determinant of 23854, and control limits ranging from 255.9327 (LCL) to 862.5890 (UCL), with a criterion (C) of 1.4471. MAEWMA has a higher mean of 726.0635, a determinant of 21304, and more expanded control limits from 439.4033 (LCL) to 1012.7 (UCL), with a C value of 1.2214, indicating larger sensitivity than the classical chart. MDEWMA reports a near-zero mean of -0.7283, a smaller determinant of 10360, and control limits between -200.6240 (LCL) and 199.1674 (UCL), with a C value of 0.4142, highlighting its tighter control. Therefore, the proposed charts can be relied upon to control and monitor the rate of the qualitative property as well as the differences (Yield point and Tensile strength) in the Erbil Iron Factory.

10. Conclusions

1. The integration of Daubechies Wavelet Transformation with the MEWMA chart shows an advanced methodology for quality control.
2. The suggested MAEWMA and MDEWMA charts exhibit advanced sensitivity to minor variations in production processes relative to conventional MEWMA charts.
3. MAEWMA Chart efficiently tracks averages using approximation coefficients, enabling effective detection of mean shifts in high-dimensional datasets.
4. The MDEWMA Chart focuses on variance monitoring via detailed coefficients, providing superior findings of changes in variability.
5. Simulation studies confirm the robustness and efficiency of these charts across different tuning parameters (λ) and sample sizes.
6. The application of data from Erbil Steel Company demonstrated the practical efficacy of the presented charts in monitoring intricate processes.

11. Recommendations

1. Implementation of Advanced Quality Control Tools: Industries characterized by intricate, high-dimensional processes ought to utilize MAEWMA and MDEWMA charts to improve quality oversight and process stability.
2. Parameter Optimization: The tuning of the parameter (λ) and wavelet order must be calibrated according to distinct process features to enhance detection sensitivity.
3. Integration with Real-Time Systems: Implement real-time monitoring systems to leverage the sensitivity of these charts for timely corrective actions.
4. Training and Awareness: Conduct training sessions for quality control teams to familiarize them with wavelet-based methodologies and their applications.
5. Further Validation: Extend the validation of these methods across various industries, including chemical, telecommunication, and biomedical fields, to ensure broader applicability.

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Conflict of Interest

The authors declare that there are no conflicts of interest regarding the publication of this manuscript.

Ethical Approval

Ethical approval was not required for this study as it did not involve human participants, personal data.

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لوحات مراقبة الجودة لمعاملات تحويل الموجات المتقطع لدوبيشيز أيوما متعدد المتغيرات مروان طارق حسن، طه حسين علي و نظيره صديق كريم

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الخلاصة: تُستخدم المتوسطات المتحركة المرجحة الأسية متعدد المتغيرات (MEWMA) للتحكم في العديد من الخصائص النوعية لعمليات الإنتاج معاً. يقترح هذا البحث إنشاء لوحات جديدة للتحكم في المتوسطات المتحركة المرجحة الأسية ذات الخصائص النوعية المتعددة المتغيرات ومراقبتها، فضلاً عن التباين من خلال تحليل الموجات المستند إلى تحويل الموجات المتقطع (Daubechies). يقسم تحليل الموجات البيانات المتعدد المتغيرات إلى معاملات تقريب وتفصيل، والتي تُستخدم لبناء لوحة المتوسطات المتحركة المرجحة الأسية لمعاملات التقريب (MAEWMA) (للتحكم في المتوسط ومراقبته) ولوحة المتوسطات المتحركة المرجحة الأسية لمعاملات التفصيل (MDEWMA) (للتحكم في التباين ومراقبته). كانت اللوحات المقترحة أكثر كفاءة من اللوحة التقليدية وأكثر حساسية للتغيرات الطفيفة التي يمكن أن تحدث في عمليات الإنتاج عند عدة قيم لمعلمة الضبط ولأحجام عينات وأعداد مختلفة من المتغيرات من خلال دراسات المحاكاة والبيانات الحقيقية.

الكلمات المفتاحية: لوحات السيطرة النوعية، لوحة أيوما متعدد المتغيرات، التحويل الموجي المتقطع، الموجة دوبيشيز، معاملات التقريب والتفصيل.