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Ranking of Intuitionistic Fuzzy Numbers by Using Scaling Method

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Abstract

Ranking intuitionistic fuzzy numbers (IFN) is a challenging task. Several methods have been presented for ranking IFNs. Also ranking for three IFN is rare. In this work, a new multidimensional scaling (MDS) method for ranking triangular intuitionistic fuzzy number (TIFN) is proposed. This method is easy to implement, visualized and embedded the (TIFN). Also, gives a possibility to configure points in different ways. Configuration points can be extracted in a two-dimensional space since each TIFN is represented as a row in a matrix. Since these points are not uniquely established, we provide a technique for reconfiguring it in order to compare it with various methods. This method is novel in sense of the idea. Lastly, the method is illustrated through numerical examples.

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1. Introduction

(Atanassov, 1986) (Atanassov, 1999) suggested intuitionistic fuzzy sets (IFS) as a generalization of Zadeh's fuzzy sets (Zadeh, 1965) (Zadeh, 1971) for modeling non-crisp and uncertain sets. He handles both membership and non-membership functions.

IFNs seem befitting for describing the uncertainty following by generalized fuzzy numbers (FN). Real numbers can be linearly ordered by the relation $\leq$ or $\geq$, however this type of inequalities do not exist in fuzzy numbers and intuitionistic fuzzy numbers. To solve this problem defined ranking function (Roseline & Amirtharaj, 2013) .

Many ranking methods for ordering of IFN have been introduced in the literature. (Roseline & Amirtharaj, 2011) introduced a new ranking method of IFN based on magnitude

In (Rezvani, 2013) proposed a new ranking method for trapezoidal IFN based on value and ambiguities index.. (Arun Prakash et al., 2016) defined centroid for membership and non-membership for both trapezoidal (TrIFN) and triangular intuitionistic fuzzy numbers respectively, and proposed a ranking method based on a centroid concept. (Bharati, 2017) proposed a ranking method to sort (TIFN) Based on fuzzy origin, estimate the distance between each intuitionistic fuzzy number and the fuzzy origin, and then compare the distances. (Mohan et al., 2020) defined the magnitude of different forms of (IFNs). In (Popa, 2023) defined ranking function, which is based on Robust's ranking index of the membership function and the non-membership function of trapezoidal intuitionistic fuzzy numbers. Also one of the recent methods for ranking IFN was based on Nagle points (Prakash & Suresh, 2024). MDS is convert data from high dimensional to lower dimensional, MDS is one

of the methods that we introduce for ranking, which is a new method first used by (Kareem & Ramadan, 2021) to rank the fuzzy numbers.
In this paper we have used the same idea with modifications for IFNs, which is done for the first time. We treated with each IFN as a row matrix to construct our matrix and used MDS technique to find the configuration points. These points represent the intuitionistic fuzzy number themselves in R2. After that we presented different techniques to reconfigure the points to compare with other methods.

1. Basic Concepts and Definitions

This section gives an overview of the basic definitions and main concepts related to intuitionistic fuzzy sets and fuzzy numbers.

Definition (1) (Atanassov, 1986) : An IFS $\tilde{A}^\iota$ in X is defined as an object of the following form $\tilde{A}^\iota = \{(x, \mu_{\tilde{A}^\iota}(x), v_{\tilde{A}^\iota}(x)): x \in X\}$, where $\mu_{\tilde{A}^\iota}: X \to [0,1]$ and $v_{\tilde{A}^\iota}: X \to [0,1]$ determine the degree of membership and the degree of non-membership of the element $x \in X$ in $\tilde{A}^\iota$, respectively and for every $x \in X, 0 \le \mu_{\tilde{A}^\iota}(x) + v_{\tilde{A}^\iota}(x) \le 1$.
For each IFN $\tilde{A}^\iota$ in , we have $\pi_{\tilde{A}^\iota}(x) = 1 - \mu_{\tilde{A}^\iota}(x) - v_{\tilde{A}^\iota}(x)$, is called the degree of non-determinacy (or uncertainty or hesitation factor) of the element $x \in X$ to the intuitionistic fuzzy set $\tilde{A}^\iota$, also $0 \le \pi_{\tilde{A}^\iota}(x)1$,for all $x \in X$.

Remark: $\pi_{\tilde{A}^\iota}(x) = 0$ then an intuitionistic fuzzy set becomes fuzzy set.

Definition (2) (Nehi, 2010) : An intuitionistic fuzzy set (IFS)$\tilde{A}^\iota = \{(x, \mu_{\tilde{A}^\iota}(x), v_{\tilde{A}^\iota}(x), x \in X)\}$ is called IF-normal, if there exist at least two points $x_1, x_2 \in X$ such that $\mu_{\tilde{A}^\iota}(x_1) = 1$, $v_{\tilde{A}^\iota}(x_2) = 1$. It is easily seen that given intuitionistic fuzzy set $\tilde{A}^\iota$is IF-normal, if there is at least one point it surely belongs to $\tilde{A}^\iota$, and at least one point does not belong to $\tilde{A}^\iota$.

Definition (3) (Nehi, 2010) :An intuitionistic fuzzy set (IFS) $\tilde{A}^\iota = \{(x, \mu_{\tilde{A}^\iota}(x), v_{\tilde{A}^\iota}(x), x \in X)\}$ of the real line is called IF-convex, if $\forall x_1, x_2 \in$ R, $\forall \lambda \in [0,1]$

$$\mu_{\tilde{A}^\iota}(\lambda x_1 + (1-\lambda)x_2) \ge \mu_{\tilde{A}^\iota}(x_1) \Lambda \mu_{\tilde{A}^\iota}(x_2)$$
$$v_{\tilde{A}^\iota}(\lambda x_1 + (1-\lambda)x_2) \ge v_{\tilde{A}^\iota}(x_1) \Lambda v_{\tilde{A}^\iota}(x_2)$$

Remark: $\tilde{A}^\iota$ is IF-convex i.e. membership function $\mu_{\tilde{A}^\iota}(x)$ is convex and non-membership $v_{\tilde{A}^\iota}(x)$ function is concave.

Definition (4) (Nehi, 2010): An intuitionistic fuzzy set (IFS) $\tilde{A}^\iota = \{(x, \mu_{\tilde{A}^\iota}(x), v_{\tilde{A}^\iota}(x), x \in X)\}$ of the real line is called IF-number IFN if:

i. $\tilde{A}^\iota$ is intuitionistic fuzzy normal
ii. $\tilde{A}^\iota$ is intuitionistic fuzzy convex
iii. $\mu_{\tilde{A}^\iota}$ is upper semi-continuous and $v_{\tilde{A}^\iota}$ is lower semi continuous.
iv. Supp $\tilde{A}^\iota = \{(\mathrm{x} \in \mathrm{X} / v_{\tilde{A}^\iota}(x) < 1)\}$ is bounded.

Definition (5) (Nagoorgani, 2012):Triangular intuitionistic fuzzy number (TIFN) is denoted by $\tilde{A}^\iota = (a_1, a_2, a_3; b_1, a_2, b_3)$,where $a_1, a_2, a_3, b_1, b_3 \in \mathbb{R}$ such that $b_1 \le a_1 \le a_2 \le a_3 \le b_3$ .whose membership and non-membership functions are defined as follows :

$$\mu_{\tilde{A}^\iota}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}; & a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}; & a_2 \le x \le a_3 \\ 0; & \text{otherwise} \end{cases} \quad \text{and} \quad v_{\tilde{A}^\iota}(x) = \begin{cases} \frac{a_2-x}{a_2-b_1}; & b_1 \le x \le a_2 \\ \frac{x-a_2}{b_3-a_2}; & a_2 \le x \le b_3 \\ 1 & \text{otherwise} \end{cases}$$

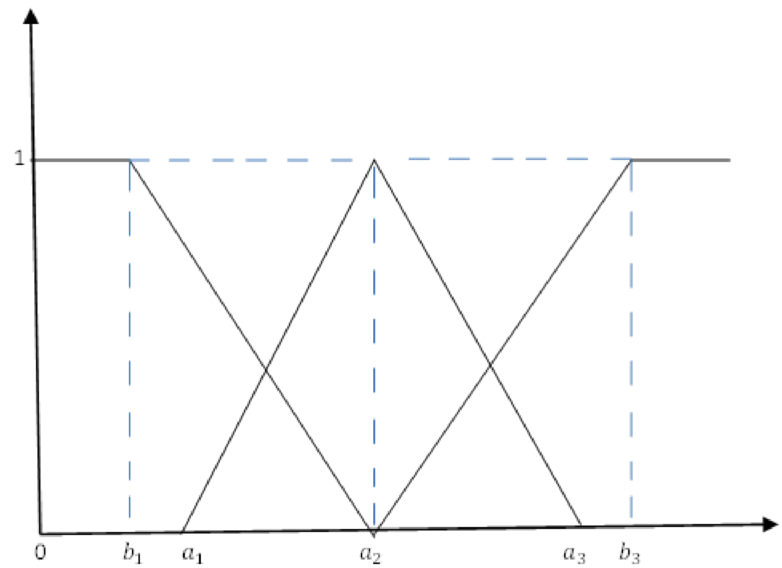

Figure (1) Triangular intuitionistic fuzzy number

2. Ranking Intuitionistic Fuzzy Numbers

Ranking intuitionistic fuzzy numbers (IFN) is an important step in decision-making, especially when dealing with ambiguity and imprecision. Several methods for ranking IFN have been proposed, including those based on magnitude and distance metrics from a fuzzy origin. Recent research has established ranking approaches specialized for several sorts of IFNs, such as triangular and trapezoidal varieties, and demonstrated their efficiency with numerical examples. Establishing trustworthy ranking algorithms is critical for applications such as linear programming and data analysis, where correct IFN comparisons can have a major impact on decision results (Bharati, 2017). One typical type of ranking strategy is to map intuitionistic fuzzy numbers directly to the real line. That is, they are transformation functions that correlate each intuitionistic fuzzy number with a real number and then apply the ordering on the real line.

3. Mathematics of Metric Multidimensional Scaling

This method (Rencher, 2009) starts with a square matrix called Euclidean matrix $D = \delta_{ij}$. The goal is to find n point in k such that the interpoint distance d_{ij} in the k dimensions are approximately equal to the value δ_{ij} in D. The points are found as follows:

1. Construct the $n \times n$ matrix $A = (a_{ij}) = (-\frac{1}{2}\delta_{ij}^2)$, where δ_{ij} is the ijth element of D.
2. Construct the $n \times n$ matrix $B = (b_{ij})$, with elements $b_{ij} = a_{ij} - \bar{a}_{i.} - \bar{a}_{.j} + \bar{a}$ ... where $\bar{a}_{i.} = \sum_{j=1}^{n} a_{ij}/n$, $\bar{a}_{.j} = \sum_{i=1}^{n} a_{ij}/n$. The matrix B can be written as

$$B = \left(I - \frac{1}{n}J\right) A \left(I - \frac{1}{n}J\right) \tag{1}$$

Since B is a symmetric matrix, we can use the spectral decomposition to write B in the form

$$B = \vee \wedge \vee' \tag{2}$$

where $\vee$ is the matrix eigenvectors of B and $\wedge$ is the diagonal matrix of eigenvalues of B.
Contains the corresponding eigenvectors, then we can express (2) in the form

$$\begin{aligned} B &= \vee_1 \wedge_1 \vee_1' \\ &= \vee_1 \wedge_1^{1/2} \wedge_1^{1/2} \vee_1' \\ &= ZZ', \end{aligned}$$

where $$Z = \mathrm{V}_1\Lambda_1^{1/2} = \left(\sqrt{\lambda_1}\mathrm{v}_1, \sqrt{\lambda_2}\mathrm{v}_2, \dots, \sqrt{\lambda_q}\mathrm{v}_\mathrm{q}\right) = \begin{pmatrix} z_1' \\ z_2' \\ . \\ . \\ . \\ z_n' \end{pmatrix}, \qquad (3)$$

So $(z_1' \ z_2' \ \dots \ z_n')^t$ is our scale points.

Note that the matrix B is not positive semi-definite. In such cases some of the eigenvalues of B will be negative; correspondingly, some coordinate values will be complex. If, however, B has only a small number of small negative eigenvalues, a useful spatial representation of the observed dissimilarity matrix may often still result from the eigenvectors associated with the K largest positive eigenvalues. If, however, B has a considerable number of large negative eigenvalues, classical scaling of the dissimilarity matrix may
not be advisable, in which case some other method of scaling-for example, non-metric scaling might be used instead.

4. Shifting Technique of Scale Points

Since the scale points $(z_1, z_2, \dots, z_n)$ belong to R^n. If all scale points in $R^+ \cup \{0\}$, do not need shifting. it is vital to notice that if the scale points are not all in $R^+ \cup \{0\}$, while calculating the Euclidean distance, which has a limitation. We apply a new shifting technique to fix the position of these points in $R^+ \cup \{0\}$. Let

$$Z = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \\ \vdots & \vdots \\ z_{i1} & z_{i2} \end{pmatrix}.$$

We present the following the preposition:

5.1. Proposition 1

If at least one of scale point less than zero, then exist $M > 0$ such that $M - z_{i1}$ and $M - z_{i1}$ are in $R^+ \cup \{0\}$.

Proof: the aim to find M. Let M_1=max $\{|z_{i1}|\}$ and M_2=max $\{|z_{i2}|\}$ $\forall i = 1,2, \dots, n$. Since $\mathrm{M}_1 > 0$ And $\mathrm{M}_2 > 0$ so let $M = \mathrm{M}_1 + \mathrm{M}_2$,its clearly $M > 0$. Therefore so, the new scale point is

$$Z^* = \begin{pmatrix} M - z_{11} & M - z_{12} \\ M - z_{21} & M - z_{22} \\ \vdots & \vdots \\ M - z_{n1} & M - z_{n2} \end{pmatrix} = \begin{pmatrix} z_{11}^* & z_{12}^* \\ z_{21}^* & z_{22}^* \\ \vdots & \vdots \\ z_{n1}^* & z_{n2}^* \end{pmatrix}$$

6.Different Techniques for Ranking

6.1. Scaling Method Z_1

In (Kareem & Ramadan, 2021) introduced a new this idea for ranking fuzzy number we applied for IFN, let $N = (z_g^*, \ z_h^*)$ such that z_g^* is maximum of z_{i1}^* and z_h^* is maximum of z_{i2}^*, for $i = 1,2, \dots, n$. Let $Z_{\tilde{A}^\iota{}_1}^* = (z_{11}^*, \ z_{12}^*)$, $Z_{\tilde{A}^\iota{}_2}^* = (z_{21}^*, \ z_{22}^*)$, $\dots, Z_{\tilde{A}^\iota{}_n}^* = (z_{n1}^*, \ z_{n2}^*)$ be the scales of the fuzzy numbers.
Calculate the distance between N and $Z_{\tilde{A}^\iota{}_i}^*$ as follows,

$$R(Z_{\tilde{A}^\iota{}_i}^*, N) = \sqrt{\left(z_g^* - z_{i1}^*\right)^2 + (z_h^* - z_{i2}^*)^2} \ .$$

If $\tilde{A}^\iota{}_1$ and $\tilde{A}^\iota{}_2$ are two arbitrary IFN then the ranking is
1. $R(Z_{\tilde{A}^\iota{}_1}^*, N) < R(Z_{\tilde{A}^\iota{}_2}^*, N)$ then $\tilde{A}^\iota{}_1 > \tilde{A}^\iota{}_2$
2. $R(Z_{\tilde{A}^\iota{}_1}^*, N) > R(Z_{\tilde{A}^\iota{}_2}^*, N)$ then $\tilde{A}^\iota{}_1 < \tilde{A}^\iota{}_2$
3. $R\left(Z_{\tilde{A}^\iota{}_1}^*, N\right) = R(Z_{\tilde{A}^\iota{}_2}^*, N)$ then $\tilde{A}^\iota{}_1 \sim \tilde{A}^\iota{}_2$.

6.2. Scaling Method Z_2

If $v = (v_1, v_2, v_3)^t$ is principal component of the matrix B, v is corresponding for maximum eigenvalue and the new scale points are $Z^*_{\tilde{A}^\iota_1} = (z^*_{11},\ z^*_{12}, 0)$, $Z^*_{\tilde{A}^\iota_2} = (z^*_{21},\ z^*_{22}, 0)$, …, $Z^*_{\tilde{A}^\iota_n} = (z^*_{n1},\ z^*_{n2}, 0)$. Then, the ranking function described as follow:

$R\left(Z^*_{\tilde{A}^\iota_i}, v\right) = \sqrt{(z^*_{i1} - v_1)^2 + (z^*_{i2} - v_2)^2 + (z^*_{i3} - v_3)^2}$, for $i = 1,2, \dots, n$

If $\tilde{A}_1$ and $\tilde{A}_2$ are two arbitrary IFN, their ranking is defined by:

1. If $R\left(Z^*_{\tilde{A}^\iota_1}, v\right) < R\left(Z^*_{\tilde{A}^\iota_2}, v\right)$ then $\tilde{A}^\iota_1 < \tilde{A}^\iota_2$
2. If $R\left(Z^*_{\tilde{A}^\iota_1}, v\right) > R\left(Z^*_{\tilde{A}^\iota_2}, v\right)$ then $\tilde{A}^\iota_1 > \tilde{A}^\iota_2$
3. If $R\left(Z^*_{\tilde{A}^\iota_1}, v\right) = R\left(Z^*_{\tilde{A}^\iota_2}, v\right)$ then $\tilde{A}^\iota_1 \sim \tilde{A}^\iota_2$.

7. Numerical Examples

In this section we solve some example by our method and Compare it with another methods.

Example (1) (Popa, 2023):

Let $\tilde{A}^\iota = (2.95, 5.1, 6.65;\ 2.25, 5.1, 9.65)$, $\tilde{B}^\iota = (3.95, 5.15, 6.2;\ 2.325, 5.15, 8.75)$ and $\tilde{C}^\iota = (3.85, 5.15, 6.4;\ 2.35, 5.15, 9.05)$ are three IFN

$$X = \begin{pmatrix} \tilde{A}^\iota \\ \tilde{B}^\iota \\ \tilde{C}^\iota \end{pmatrix} = \begin{pmatrix} 2.95 & 5.1 & 6.65 & 2.25 & 5.1 & 9.65 \\ 3.95 & 5.15 & 6.2 & 2.325 & 5.15 & 8.75 \\ 3.85 & 5.15 & 6.4 & 2.35 & 5.15 & 9.05 \end{pmatrix}$$

The D matrix is

$$D = \begin{pmatrix} 0 & 1.4224 & 1.1169 \\ 1.4224 & 0 & 0.3750 \\ 1.1169 & 0.3750 & 0 \end{pmatrix}$$

The A matrix is

$$A = \begin{pmatrix} 0 & -1.0116 & -0.6237 \\ -1.0116 & 0 & -0.0703 \\ -0.6237 & -0.0703 & 0 \end{pmatrix}$$

The B matrix is

$$B = \begin{pmatrix} 0.7112 & -0.4849 & -0.2263 \\ -0.4849 & 0.3422 & 0.1426 \\ -0.2263 & 0.1426 & 0.0837 \end{pmatrix}.$$

Find eigenvalues and eigenvectors of B such that $\lambda_{1=}1.1148$, $\lambda_{2=}0$ and $\lambda_{3=}0.0223$. The eigenvector corresponding eigenvalues are

$$v_1 = \begin{pmatrix} -0.7984 \\ 0.5474 \\ 0.5474 \end{pmatrix}, v_2 = \begin{pmatrix} 0.5774 \\ 0.5774 \\ 0.5774 \end{pmatrix} \text{ and } v_3 = \begin{pmatrix} -0.1711 \\ -0.6058 \\ 0.7770 \end{pmatrix},$$

then

$$Z = \left(\sqrt{\lambda_1}v_1, \sqrt{\lambda_3}v_3\right) = \begin{pmatrix} -0.8429 & -0.0256 \\ 0.5780 & -0.0905 \\ 0.2650 & 0.1161 \end{pmatrix}.$$

We must shift by proposition (1), $m_1 = 0.8429$ and $m_2 = 0.1161$ so, $M = 0.9590$ then

$$Z^* = \begin{pmatrix} 1.8019 & 0.9846 \\ 0.3810 & 1.0495 \\ 0.6940 & 0.8429 \end{pmatrix}$$

Ranking by Z_1, $N = (1.8019, 1.0495)$, $R(Z^*_{\tilde{A}^\iota}, N) = 0.0649$, $R(Z^*_{\tilde{B}^\iota}, N) = 1.4209$, $R(Z^*_{\tilde{C}^\iota}, N) = 1.1270$. therefore, $\tilde{B}^\iota < \tilde{C}^\iota < \tilde{A}^\iota$. By Z_2, $v = v_1^t = [-0.7984, 0.5474, 0.5474]$, $R(Z^*_{\tilde{A}^\iota}, v) = 2.6930$, $R(Z^*_{\tilde{B}^\iota}, v) = 1.3938$, $R(Z^*_{\tilde{C}^\iota}, v) = 1.6169$. therefore, $\tilde{B}^\iota < \tilde{C}^\iota < \tilde{A}^\iota$.

Example (2):

In this Example, we will apply our proposed ranking methods on some cases of IFN.

Set 1: $\tilde{A}^\iota = (2.95, 5.1, 6.65;\ 2.25, 5.1, 9.65)$, $\tilde{B}^\iota = (3.95, 5.15, 6.2;\ 2.325, 5.15, 8.75)$ and $\tilde{C}^\iota = (3.85, 5.15, 6.4;\ 2.35, 5.15, 9.05)$

Set 2: $\tilde{A}^\iota = (1.1, 1.5, 1.8; 1.05, 1.5, 1.85)$, $\tilde{B}^\iota = (1.2, 1.48, 1.78; 1.07, 1.48, 1.8)$ and $\tilde{C}^\iota = (1.3, 1.4, 1.7; 1.01, 1.4, 1.9)$

Set 3: $\tilde{A}^\iota = (4.0, 5.05, 4.6;\ 3.2, 5.05, 6.3)$, $\tilde{B}^\iota = (3.5, 5.0, 4.5; 3.0, 5.0, 6.0)$ and $\tilde{C}^\iota = (3.8, 5.05, 4.4;\ 3.1, 5.05, 6.1)$

Set 4: $\tilde{A}^\iota = (13.2, 13.1, 13.8, 11.1, 13.1, 15.0)$, $\tilde{B}^\iota = (12.5, 13.0, 14.0, 11.0, 13.0, 15.5)$ and $\tilde{C}^\iota = (12.8, 12.9, 13.5, 11.5, 12.9, 14.8)$

The results of the proposed methods are shown in the following Tables

Table 1: Results for the (set 1, set 2, set 3, set 4)

Authors	**IFN**	**Set 1**	**Set 2**	**Set 3**	**Set 4**
Rezvani	$\tilde{A}^\iota$	5.1917	1.4833	4.8750	13.1583
	$\tilde{B}^\iota$	5.2021	1.4742	4.7500	13.0880
	$\tilde{C}^\iota$	5.2375	1.4252	4.8167	12.9833
Results		$\tilde{A}^\iota < \tilde{B}^\iota < \tilde{C}^\iota$	$\tilde{C}^\iota < \tilde{B}^\iota < \tilde{A}^\iota$	$\tilde{B}^\iota < \tilde{C}^\iota < \tilde{A}^\iota$	$\tilde{C}^\iota < \tilde{B}^\iota < \tilde{A}^\iota$
Roseline and Amirtharaj	$\tilde{A}^\iota$	5.05	1.4917	4.9250	13.1667
	$\tilde{B}^\iota$	5.1375	1.4817	4.9333	13.0417
	$\tilde{C}^\iota$	5.1458	1.4167	4.8917	12.9412
Results		$\tilde{A}^\iota < \tilde{B}^\iota < \tilde{C}^\iota$	$\tilde{C}^\iota < \tilde{B}^\iota < \tilde{A}^\iota$	$\tilde{B}^\iota < \tilde{C}^\iota < \tilde{A}^\iota$	$\tilde{C}^\iota < \tilde{B}^\iota < \tilde{A}^\iota$
Prakash et al.	$\tilde{A}^\iota$	5.0913	1.4333	4.1017	12.7044
	$\tilde{B}^\iota$	4.8847	0.9741	3.8370	12.7503
	$\tilde{C}^\iota$	5.0127	0.9837	3.936	12.6503
Result		$\tilde{B}^\iota < \tilde{C}^\iota < \tilde{A}^\iota$	$\tilde{A}^\iota < \tilde{B}^\iota < \tilde{C}^\iota$	$\tilde{B}^\iota < \tilde{C}^\iota < \tilde{A}^\iota$	$\tilde{C}^\iota < \tilde{A}^\iota < \tilde{B}^\iota$
Bharati	$\tilde{A}^\iota$	2.6187	1.2906	0.9266	2.0288
	$\tilde{B}^\iota$	1.8309	1.1689	1.5417	4.0288
	$\tilde{C}^\iota$	2.01	0.6466	0.9420	2.7629
Results		$\tilde{B}^\iota < \tilde{C}^\iota < \tilde{A}^\iota$	$\tilde{C}^\iota < \tilde{B}^\iota < \tilde{A}^\iota$	$\tilde{A}^\iota < \tilde{C}^\iota < \tilde{B}^\iota$	$\tilde{A}^\iota < \tilde{C}^\iota < \tilde{B}^\iota$

Mohan et al.	$\tilde{A}^\iota$ $\tilde{B}^\iota$ $\tilde{C}^\iota$	5.3333 5.2667 5.3292	1.4750 1.4667 1.4350	4.8250 4.6670 4.7417	13.1500 13.1250 13.0250
Results		$\tilde{B}^\iota < \tilde{C}^\iota < \tilde{A}^\iota$	$\tilde{C}^\iota < \tilde{B}^\iota < \tilde{A}^\iota$	$\tilde{B}^\iota < \tilde{C}^\iota < \tilde{A}^\iota$	$\tilde{C}^\iota < \tilde{B}^\iota < \tilde{A}^\iota$
Popa	$\tilde{A}^\iota$ $\tilde{B}^\iota$ $\tilde{C}^\iota$	$\tilde{A}^\iota < \tilde{B}^\iota < \tilde{C}^\iota$ $\tilde{B}^\iota < \tilde{A}^\iota < \tilde{C}^\iota$ $\tilde{B}^\iota < \tilde{C}^\iota < \tilde{A}^\iota$	$\tilde{C}^\iota < \tilde{B}^\iota < \tilde{A}^\iota$ $\tilde{C}^\iota < \tilde{A}^\iota < \tilde{B}^\iota$	$\tilde{C}^\iota < \tilde{B}^\iota < \tilde{A}^\iota$	$\tilde{C}^\iota < \tilde{A}^\iota < \tilde{B}^\iota$ $\tilde{C}^\iota < \tilde{B}^\iota < \tilde{A}^\iota$
Results	Dependent on λ	Dependent on λ	Dependent on λ	Dependent on λ	Dependent on λ
Z_1 method	$\tilde{A}^\iota$ $\tilde{B}^\iota$ $\tilde{C}^\iota$	0.0649 1.4209 1.1270	0 0.1187 0.2722	0.00016 0.6285 0.3613	0.7520 0.4945 1.2326
Results		$\tilde{B}^\iota < \tilde{C}^\iota < \tilde{A}^\iota$	$\tilde{C}^\iota < \tilde{B}^\iota < \tilde{A}^\iota$	$\tilde{B}^\iota < \tilde{C}^\iota < \tilde{A}^\iota$	$\tilde{C}^\iota < \tilde{A}^\iota < \tilde{B}^\iota$
Z_2 method	$\tilde{A}^\iota$ $\tilde{B}^\iota$ $\tilde{C}^\iota$	2.6930 1.3938 1.6169	1.2825 1.1996 1.0977	1.4713 0.8545 1.2007	2.0740 2.5774 1.5354
Results		$\tilde{B}^\iota < \tilde{C}^\iota < \tilde{A}^\iota$	$\tilde{C}^\iota < \tilde{B}^\iota < \tilde{A}^\iota$	$\tilde{B}^\iota < \tilde{C}^\iota < \tilde{A}^\iota$	$\tilde{C}^\iota < \tilde{A}^\iota < \tilde{B}^\iota$

In Table 1 and set 1, our methods are capable to recognize these numbers, Z_1 and Z_2 give us $\tilde{B}^\iota < \tilde{C}^\iota < \tilde{A}^\iota$, like some methods. For set 2, they give this result $\tilde{C}^\iota < \tilde{B}^\iota < \tilde{A}^\iota$, where as some of other methods. For set 3, they give this result $\tilde{B}^\iota < \tilde{C}^\iota < \tilde{A}^\iota$, where as some of other methods. For set 4, they give this result $\tilde{C}^\iota < \tilde{A}^\iota < \tilde{B}^\iota$, where as some of other methods. The method was implemented in MATLAB 7, and run on a 3.00 GHz Laptop with 16.0 GB. memory

8. Conclusion

Ranking three IFN is difficult because of existing two factors, membership function and non-membership function. In this investigation, we define a new ranking function for ordering TIFN which is MDS as a base. The concept is new for IFN and has a simple structure. It embedded the number on R^2 in such a way to keep the distance between the numbers nearly the same. This method convert the numbers to points, then used some shifting to make the pints positive. The method shows capability to rank different types of numbers. Also, the comparison gives the same results with other methods.

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Conflict of interest

The author has no conflict of interest.

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تصنيف الأرقام الضبابية البديهية باستخدام أسلوب القياس.

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الخلاصة: إن تصنيف الأعداد الضبابية الحدسية (IFN) مهمة صعبة. وقد تم تقديم العديد من الطرق لتصنيف الأعداد الضبابية الحدسية. كما أن التصنيف لثلاثة أعداد ضبابية حدسية أمر نادر. في هذا العمل، تم اقتراح طريقة جديدة للقياس متعدد الأبعاد (MDS) لتصنيف الأعداد الضبابية الحدسية المثلثية (TIFN). يمكن استخراج نقاط التكوين في مساحة ثنائية الأبعاد حيث يتم تمثيل كل عدد ضبابي حدسي مثلثي كصف في مصفوفة. ونظرًا لأن هذه النقاط ليست محددة بشكل فريد، فإننا نقدم تقنية لإعادة تكوينها من أجل مقارنتها بطرق مختلفة. يتم توضيح هذه الطريقة من خلال الأمثلة العددية.

الكلمات المفتاحية: الأرقام الضبابية الحدسية المثلثية، الترتيب، التدرج متعدد الأبعاد، نقاط التكوين.