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A Multi-Objective Optimization Approach for Design Earth-Fill Dams Variables

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Abstract

The entire research study is about the optimization of design variables for an earth-fill dam utilizing multi-objective optimization approach. We have considered three variables that are related to the soil material properties which are core density, foundation Young's modulus of elasticity, and shell coefficient of permeability. A limited range of each variable is applied depending on the literature and numerical simulations are dedicated for the response of the global structure using ABAQUS program. Box-Behnken design method along with MATLAB codes with least-square method are being used to construct two surrogate models for the displacements of the earth-fill dam. Fifteen numerical models are involved in the process with the presence of non-linear equations for the objective function for the optimization process. The objective functions were for the pore water pressure and the maximum principal stress were they would be first checked for reliability by the coefficient of determination check R^2. The reliability of the objective function was 100% which enhanced them to be ready for the multi-objective optimization step. The results of the optimized variables for both objective functions were determined and compared with the minimal responses of the considered models of the numerical simulations of the global structure. The optimum results of both objective functions of the earth-fill dam were determined and approved which is an indication of excellent result for the optimization of the design variables.

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1. Introduction

The world is currently facing climate change due to increased population, factories and transportation, which has directly affected the amount of water, causing water shortages and declining groundwater levels. Building dams is the best way to combat these threats. Oldest dam still in use is a rock-fill embankment about 6 meters (20 feet) high on the Orontes River in Syria, built about 1300 BCE for local irrigation use.

Earth fill dams can be designed with a variety of structures and cross sections. An earth fill dam consists of suitable soils obtained through excavation from another area and then spread and compacted in layers by mechanical means (Khuri & Mukhopadhyay, 2010). There are several types of fill dams that can be constructed, including level dams, zoning dams, hydraulic dams, and modified dams. Earth fill dams are the most useful. This is because earth dams are typically less expensive than gravity dams, which are typically built with large amount of concrete (Ferreira et al., 2007). Due to lack of attention to scientific research before the construction of dams, and lack of scientific methods for the causes of failure of dams, many dams failed from the beginning of their construction or due to earthquakes (Mahmood et al., 2022). We have several examples of dams that have been failed such as the destroyed Chaq-Chaq dam which was one of the small core clay body dams that was constructed about 2 km northwest to Sulaimani city in Kurdistan region – Iraq. This dam was especially needed for flood control, irrigation and navigation in the region. The dam collapsed on 4th of February, 2006 (Mohammed & Baban, 2024).

Earth-fill dam are mostly exposed to seepage problem due to bad design of materials or bad construction on site. What is important to us in this study is the design of the soil material property variables which are playing the vital roles in the efficiency of the structure against seepage case. Many intensive and wide researches were adopted by designers to control the problem of the seepage in earth-fill dam constructions. (Kanchana & Prasanna, 2015) adopted the analysis of many materials for the core of earth dams with impervious property. They studied the phreatic line in the downstream through changing the horizontal drainage filters. The detected that the considered three methods resulted in a compatible form of the seepage line for the physical model. Also, they revealed a certain efficient combination of the silt and the soil materials to control the seepage. (Khassaf & Madhloom, 2017) discussed the seepage effect on the earth-fill dam where piping operation results in a sudden failure. He studied the change of core material and thickness and their effects on the seepage problem represented by many aspects such as gradient exit, amount of seepage, pressure head, and hydraulic gradient. (Zahedi & Aghazani, 2018) studied the problem of seepage by considering three shapes of the core utilizing numerical simulations for the seepage problem. They used different heights for the earth-fill dam with inclined, vertical, and diaphragm core types. (Majeed, 2015) studied the simulation of seepage of an earth-fill dam. (Jairry, 2010) applied finite element method to analyze the prediction of 2D steady state seepage in an earth-fill dam with two soil zones over a base which impervious. (Choi et al., 2016) evaluated the problem of seepage in the earth-fill dam by the utilization of 3D models of numerical simulation by finite element program. (Jamel, 2018) investigated the seepage magnitude in a homogenous core of an earth-fill dam using a finite e element program. (Fattah et al., 2014) analyzed the seepage of an earth-fill dam using finite element program through numerical models. They studied many different variables such as the permeability of embankment materials and impervious cover property in addition to the thickness and location of the core zone.

In order to control the design of the earth-fill dam, we need to undergo optimization approach for the design variables which are the most sensitive and deciding. We have used the multi-objective optimization method to control the permeability and failure of an earth-fill dam. Three variables (properties of the soil material) are considered for the optimization approach. The Box-Behnken design sampling method is being used to prepare 15 numerical models by dedicating ABAQUS finite element program. Surrogate model equations are determined for the maximum principal stress and the pore water pressure. The efficiency of the objective functions would be calculated and then would be prepared for the multi-objective optimization process.

2. Box-Behnken Design Method

Box-Behnken designs method is utilized to create higher order surrogate models considering fewer required runs than a normal factorial method. The Box-Behnken design needs twelve middle edge nodes and three central nodes to meet a nonlinear second order equation. The central composite with Box-Behnken gives a full factorial with additional three extra samples assigned at the center. Box-Behnken designs put many points on the middle of the edges of the design region which is a cube in addition to points at the center. This method authorizes a tool for the surrogate model by preparing the samples for the vector of the main matrix. The models created in the method are locating in the middle of the cubic region with the dimension $k-1$. When the problem has three variables, then the points are positioned in the middle part of the edges throughout the domain (see Figure 1).

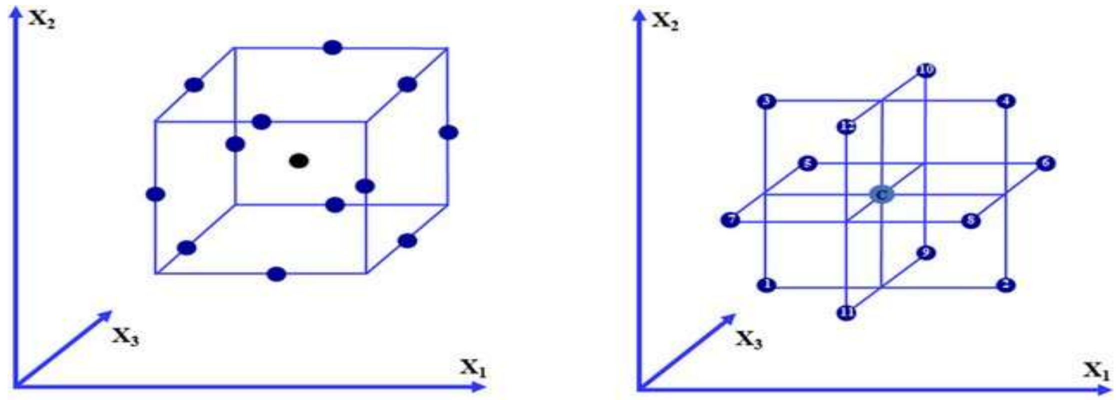


Figure 1. Box-Behnken Design Method (Three Variables) (Ferreira et al., 2007; Mahmood et al., 2022)

3. Surrogate Model

The surrogate model equation of prediction is established by considering three variables, which are dam core density, dam foundation Young's modulus of elasticity, and dam shell coefficient of permeability. The surrogate model equations for the pore water pressure and the soil maximum principal stress are authorized as follows:

$$y = f(x)\beta + \epsilon \tag{1}$$

where x is a vector starting from $1, \dots, k$ with a function $f(x)$ of k elements. β is a regression coefficients vector, and ϵ is with zero mean which is random error. The surrogate model equation requires determining the regression coefficients which are represented by β which is obtained by:

$$\beta = (X'X)^{-1}X'y \tag{2}$$

where X' is the transpose of X, and $(X'X)^{-1}$ is the inverse of $X'X$ (Choi, et al., 2016). The function $f(x)$ for both water pore pressure and the soil maximum principal stress of the earth-fill dam compromise linear, quadratic and interaction terms for the involving three variables. The equation which represents the surrogate model would be checked for reliability using the coefficient of determination which is denoted by R^2. The output of the equation would be compared with the output of the simulation and the percentage of error would be determined. It is worthy to mention that MATLAB codes are being used in determining this value for both surrogate models.

4. Finite Element Method

The finite element model of the earth-fill dam is created in 2D and the structure consists of three parts: Foundation, Shell, and Core. The foundations has the dimensions (340* 50) m and the Shell in both sides has the dimensions (110*50) m while the core is (30) m in the bottom and (10) m in the top with (50) m height as shown in Figure 2.

The material of the dam is soil with different material properties up-to foundation, shell and core parts. The model has been assigned to a soils step with duration of 1 second including creep, swelling and viscoelastic behavior with a gravity load assigned to the global model. A displacement boundary condition was given to the dam with pore pressure type boundary to simulate the pore water pressure and the saturation in the structure.

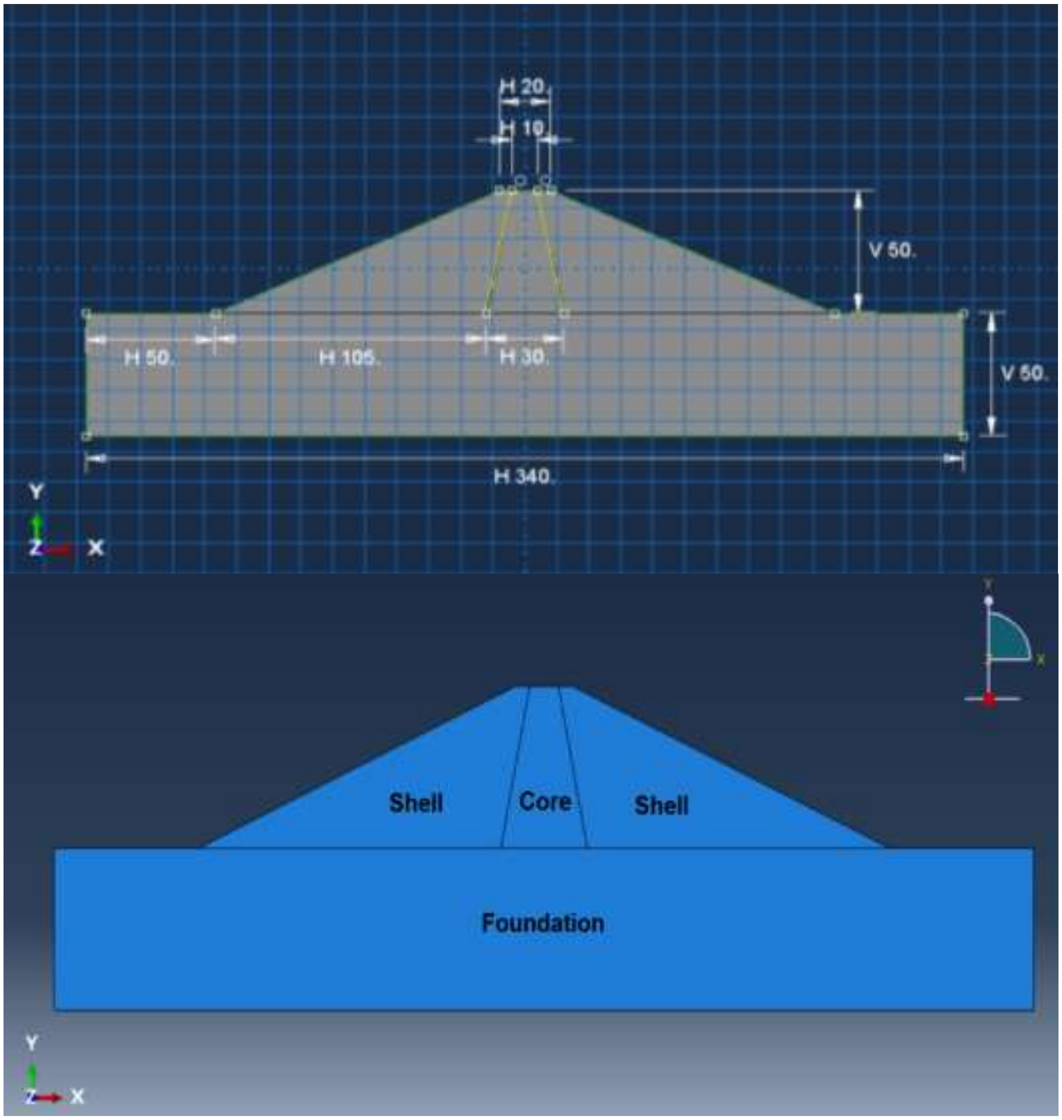


Figure 2. Finite Element Model (Earth-Fill Dam)

5. Material Data

The surrogate model equations are constructed depending on the three variables considering the Box-Behnken design sampling approach by dedicating 15 models of the earth-fill dam in ABAQUS program. The models are simulated up-to the designed samples, and both the pore water pressure and the soil coefficient of permeability are determined. Table 1 displays the considered variables and their limited values.

Table 1. Variables and Range.

Variable Symbol	Variable Name	Range
X_1	Dam Core density (ρ) kg/m^3	1700–1800
X_2	Dam Foundation Young's modulus of elasticity (E) GPa	1–1.4
X_3	Dam Shell Coefficient of Permeability (k) unitless	0.00012–0.00038

6. Multi-Criteria Optimization Problems

Classical optimization methods for such problems cannot be applied, because the numbers of variables are too many, as well as the type of the objective function which is mostly non-linear. Quadratic constrained programs (QCPs) are an important class of optimization problems with diverse real-world engineering applications (Zhou et al., 2024). So, a multi-objective linear program (MOLP) is linear programming that contains more than one objective function, which is a special case of vector linear programming (Nahar et al., 2023). Here a multi-objective optimization model is established to explore the optimum values for design variables. The mathematical formulation is as follows:

$$\left.\begin{array}{ll} \text{Max } f_i = C_i^T X + \frac{1}{2} X^T G_1 X & i = 1, ..., m \\ \text{Min } f_i = C_i^T X + \frac{1}{2} X^T G_1 X & i = m+1, ..., m_1 \\ X^L \leq X \leq X^U & X \geq 0, \end{array}\right\} \quad (3)$$

X denotes the design variable vector. X^L and X^U denote upper bounds and lower bounds, respectively. n denotes the dimension of X. f_i $(i = 1, \ldots, m_1)$ denotes the objective functions where m is number of objectives to be maximized, $(m_1 - m)$ is number of objectives to be minimized. C is n-dimensional vector, G is (n x n) a matrix of coefficients with G is a symmetric matrix (Meng et al., 2024). Optimization techniques have a great role to treat with this type of optimization (Nariman et al., 2021, 2023). Our problem contains two quadratic objectives with minimum cases, 3 variables, and 3 constraints (see Appendix 1). We start with solving each objective individually with the same constraints. After that the multi objective linear programming problem can be converted into a single objective function by various methods (Hussein, 2024). Harmonic means is one of the powerful techniques for this purpose (Ramadan, 2023).

Let HA of a set of data is defined as the reciprocal of the arithmetic average of the reciprocal of the given values. If $(x_1, x_2, \ldots, x_n)$ are n observations, then:

$$HA = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}} \quad (4)$$

To combine them, we find the common set of the variables from the following combined objective functions. Let $Min. f_i = O_i, i = 1, \ldots, m$ and $Max. f_i = O_i, i = 1 + 1, \ldots, m_1$. To formulate the problem to single objective and by using harmonic mean we have:

$$Max. g = \sum_{k=1}^{m} \frac{Max. f_k}{HA} - \sum_{k=r+1}^{m_1} \frac{Min. f_k}{HA_1} \quad (5)$$

where HA and HA_1 are the harmonic mean for maximized and minimized objectives, respectively. $Min. g$ is the combined criteria (Ramadan, 2023) . For our problem:

$$Max. g = -\sum_{k=r+1}^{m_1} \frac{Min. f_k}{HA_1} = \quad Min. g = \sum_{k=r+1}^{m_1} \frac{Min. f_k}{HA_1} \quad (6)$$

6.1 Optimized Numerical Models

The optimized models have been identified by using MATLAB codes, the results are as follows:

Pore Water Pressure $= -47.992940572719192$, Maximum Principal Stress $= 3.798127788844968e-2$. So the harmonic mean is $HA_1 = 7.596255577689936e-21$. Now, divide the coefficients of each objective functions by HA_1, and then sum them.

Depending on the optimized results which have been calculated for the three variables for both cases are:

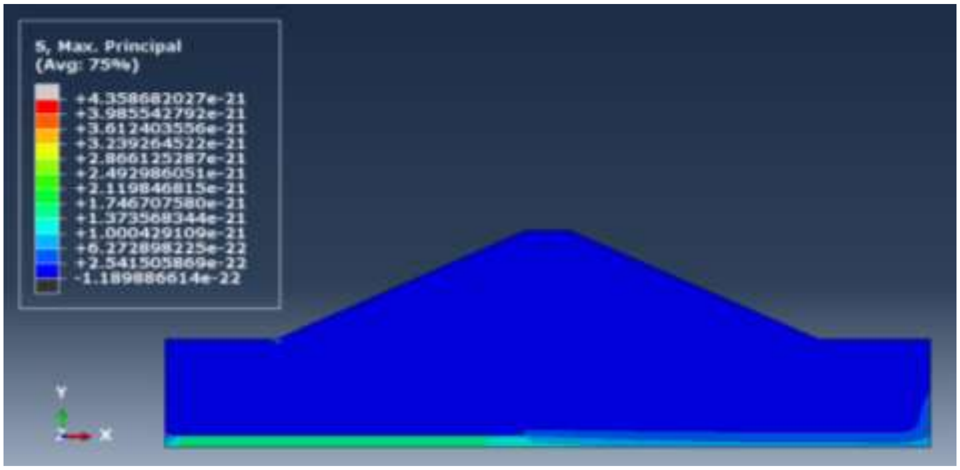
$X_1 = 1.7245e+03$, $X_2 = 1.0000e+09$, and $X_3 = 3.8000e-04$, where the magnitudes of the design variables fall in the exact range imposed. Now each output would be calculated and the results of the objective functions are:

Minimum of Maximum Principal Stress $= 4.259326922713927e-21$.

Minimum of Pore Water Pressure $= 9.293512446085509e+05$.

6.1.1 Maximum Principal Stress

The minimum result for the output maximum principal stress among the involved 15 models was $(4.358682027e-21)$, see the following Figure 3. When compared to the minimum results calculated from the optimization approach which was $(4.259326922713927e-21)$, we discover that the process was successful and we could determine minimum optimized variables to control the maximum principal stress in the earth-fill dam as the output or the first objective function.



6.1.2 Pore Water Pressure

The minimum pore water pressure for the earth-fill dam calculated from the numerical simulation was $(9.293571875e + 5)$, see Figure 4. In the other side, the optimized result for the mentioned objective function was $(9.293512446085509e + 05)$. The result is evidence that the optimization approach came out with a strong result for the earth-fill dam behavior. Also, the three design variables are optimized successfully to prepare the surrogate models for the multi-objective optimization.

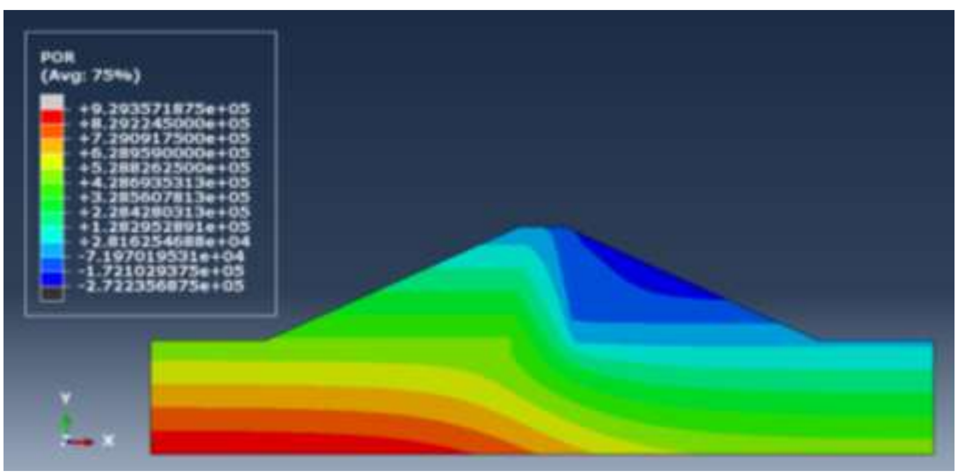


Figure 4. Finite Element Model (Minimum) Pore Water Pressure

We can see successful results of the optimization and the numerical simulations of the 15 models of the earth-fill dam. These results are a firm base for the future to consider more outputs or objective functions to optimize and then controlling the design as wanted by engineers.

7. Conclusions

Supporting on the results of the optimization process and the numerical simulation, we have concluded the following points:

1- The Considered design sampling method manifested a great efficiency in the surrogate models constructed for both objective functions. The results were verified using coefficient of determination to assure the prediction of the results for both cases up to 100% approximately.

2- The ABAQUS finite element program resulted in real simulation for the behavior of the earth-fill dam for both outputs. The 15 models were exactly authorizing the optimized results which have a very good indication for the design control by the involved three variables.
3- MATLAB codes displayed highest ability in controlling the surrogate models for the objective functions by approximating the efficiency.
4- The earth-fill dam design variables can be easily optimized with a shortest duration and minimum computational cost which is a great benefit for design engineers. Consequently, the design of the earth-fill dam structures can be controlled for any objective function in the future.

Appendix 1:

$$f_1 = 0.00000312509605483930824951352075*X_1^2 - 0.00000000000156248363495815304244332581*X_1*X_2 -$$
$$0.00000411418877626863171151827200*X_1*X_3 - 0.00921591766007444877784049452*X_1 +$$
$$0.00000000000000000019531344851293987234*X_2^2 - 0.00000000000035058494072002301013048230*X_2*X_3 +$$
$$0.0000000023047277657809544850126515*X_2 + 251017015.36707517504692077*X_3^2 -$$
$$203873.90712593961507081985*X_3 + 29399.26342694950290024280$$

$$f_2 = 0.000000000000000000000000000010763749938918432928445058029*X_1^2 +$$
$$8.0571762429220551235860005844e-44*X_1*X_2 -$$
$$0.00000000000000000000011669507694958225117169738276*X_1*X_3 +$$
$$0.000000000000000000000002059587494212325809 1*X_1 - 3.1933437379524235007827735808 03e^{-43}*X_2^2$$
$$+ 0.000000000000000000000000000024128951921152004197238127591*X_2*X_3 -$$
$$3.6974534177621977402823124971e^{-33}*X_2 - 0.000000000000000094651954880592701164744742679*X_3^2 +$$
$$0.000000000000000000060176143107788962648627794955*X_3 +$$
$$0.0000000000000000000006549028318361666743876697 1683$$

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تقريب تحسين دوال متعددة الأهداف لتصميم متغيرات السدود الترابية

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الخلاصة: تبحث الدراسة بأكملها حول تحسين متغيرات التصميم لسد ترابي باستخدام نهج التحسين متعدد الأهداف. لقد أخذنا في الاعتبار ثلاثة متغيرات مرتبطة بخصائص مادة التربة وهي كثافة اللب ومعامل مرونة يونغ للأساس ومعامل نفاذية الغلاف. يتم تطبيق نطاق محدود لكل متغير اعتمادًا على الأدبيات ويتم تخصيص المحاكاة الرقمية لاستجابة الهيكل العالمي باستخدام برنامج ABAQUS. يتم استخدام طريقة تصميم Box–Behnken جنبًا إلى جنب مع أكواد MATLAB مع طريقة المربعات الصغرى لبناء نموذجين بديلين لإزاحات السد الترابي. تشارك خمسة عشر نموذجًا رقميًا في العملية مع وجود معادلات غير خطية لدالة الهدف لعملية التحسين. كانت الدوال الهدف لضغط الماء في المسام والحد الأقصى للإجهاد الرئيسي حيث سيتم فحصها أولاً للتأكد من موثوقيتها من خلال معامل تحديد الفحص R^2. كانت موثوقية دالة الهدف 100٪ مما عززها لتكون جاهزة لخطوة التحسين متعدد الأهداف. تم تحديد نتائج المتغيرات المحسنة لكلا الدالتين الهدفيتين ومقارنتها بالاستجابات الدنيا للنماذج المدروسة للمحاكاة العددية للهيكل العالمي. تم تحديد واعتماد النتائج المثلى لكلا الدالتين الهدفيتين لسد الردم الترابي وهو مؤشر على نتيجة ممتازة لتحسين متغيرات التصميم.

الكلمات المفتاحية: نموذج بديل؛ مصطلح التفاعل؛ تصميم بوكس– بهينكن؛ ضغط الماء في المسام؛ أقصى إجهاد رئيسي، البرمجة التربيعية.