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Proposed Quality Control Charts Using Haar Wavelet Coefficients for Enhanced Production Monitoring

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Abstract

One main problem of the traditional quality control charts, such as the Individual Observations Chart and the Moving Average Chart, is that they do not focus on monitoring the differences in the produced materials. To address this issue, researchers suggested creating new charts based on the Haar wavelet that could potentially put more focus and better handle the data noise affecting traditional charts' accuracy. The new proposed charts are based on a method called wavelet transform for Haar wavelet. One chart records the average of individual observations (Approximate coefficients or low pass filter) while the other monitors the variations among these observations (Detail coefficients or high pass filter). For the first time, the universal threshold method to treat data noise was used to create control limits in the proposed charts. The researchers used both simulated and real data to develop these charts using MATLAB software. The study proved the accuracy and efficiency of the proposed charts, their success in handling the data noise, and their sensitivity in detecting minor changes that may occur in the production process.

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1. Introduction

In the early 1920s, the field of quality control witnessed a significant improvement with the introduction of the Shewhart Control Chart by Walter A. Shewhart. His invention allowed manufacturers to monitor and improve process quality through a systematic method and by visually tracking data points over time (Barlow and Irony, 1992). The Shewhart Control Chart was an initiative to distinguish between variations of a common cause (ordinary fluctuations within a process) and variations of a special cause (indicating potential problems), which allowed organizations to use accurate statistical data to make informed decisions (Ali et al. 2024).

After Shewhart, key figures like Edwards Deming and Joseph Juran made great contributions in the 1950s and 1960s in popularizing the principles of quality management in manufacturing, which led to continuous improvement and customer satisfaction. The influence of such scientists led to the adoption of quality control practices across different industries worldwide (Zabell, 1992; Bergman, 2009).

Later in the 1980s, new technologies were invented that led to the incorporation of advanced analytical methods with traditional statistical methods. This period marked the introduction of Statistical Process Control (SPC), which employed control charts to be more effective in the monitoring of manufacturing processes. The integration of computers into quality control systems enabled real-time data analysis, which enhanced the ability to detect variations and ensured adherence to quality standards (Samed et al. 2024).

In the 1990s, Discrete Wavelet Transform (DWT) was introduced and emerged as an effective tool for multiresolution analysis (Ali et al. 2024). This allowed for the examination of signals both in frequency and time domains at the same time. Because of its binary nature, the Haar wavelet was perfect for detecting abrupt changes in data, but its use was limited in fields such as image processing. By the early 2000s, DWT techniques started to be used in industrial quality control and were recognized to be superior to traditional methods. Researchers showed that Haar wavelet DWT could be effective in identifying small and sudden shifts or anomalies in production, which was crucial for the enhancement of sensitivity to changes (Mustafa and Ali, 2013). This was a feature that the traditional methods lacked. As it was important for industries to use sophisticated monitoring tools, wavelet-based methods started to be used more frequently in sectors like pharmaceuticals, electronics, and automotive manufacturing. Moving into the 2010s, Haar wavelet DWT became more integrated into advanced quality control systems. In terms of collecting detailed production data, the advancements in sensor technology made it much easier and that enabled real-time monitoring solutions. Studies proved that wavelet-based quality monitoring systems were more effective in the process of detecting critical deviations in production quality compared to traditional methods (Abramovich et al. 2000).
In 2024, researchers (Sakar et al. 2024) proposed single-valued charts based on the wavelet shrinkage of the Daubechies wavelet to handle data noise. In the same year, researchers (Duaa et al. 2024) presented the CUSUM chart based on the wavelet shrinkage of the Symlets wavelet, which was more efficient than classical charts and addressed the problem of outliers and noise.
This article proposes charts based on Haar wavelet analysis. Two charts are created, one for controlling the Haar approximation coefficients (mean) and the other for controlling the Haar detail coefficients (difference or variance). It is not available in traditional single charts, as well as addressing the noise problem using low and high pass filters.

2. Quality Control Charts

Charts for Quality Control are to determine if a process is under or out of control and to help make informed decisions about how the process is progressing through all its stages. They are regarded as one of the most popular statistical methods in the field of quality control of the production and service process (Ali, 2007).
Qualitative Control Chart Types The two primary categories of control charts are based on the kind of data that is handled by the production or service process. They're Quality Control Charts for Variables Qualities Control Chart Attributes Control Diagrams for Characteristics fundamental elements of control charts the fundamental design of control charts was created by scientist Shewhart and consists of the control chart, which is the middle boundary (Koetsier et al. 2012). Central Control Limit (CCL) Central Control Limit is one possibility. As an illustration, it shows the average It shows the control chart's upper limit, which is represented by the following: (Upper Control Limit (UCL) Upper Control Limit 2. The process is out of control, exceeding what is allowed and deviating three standard deviations from the central control limit (Ali and Esraa 2017). It symbolizes the control chart's lower limit, which is as follows: (Lower Control Limit (LCL) Lower Control Limit 3 The process measurements are three standard deviations outside of the central control limit and cannot be less than it.

3. Individual Control Chart

The individual control chart is a tool that is used to track variable data. It consists of two charts: one shows the results of individual samples (X), and the other displays the moving range (R) between these samples. This chart is especially helpful for monitoring processes where data is not collected too often (Ali et al. 2017). It looks at how individual sample results change over time. Since we do not use rational subgrouping here, it is important to think about when the results would be measured. If the process is stable, the average on the ‘individuals’ chart gives a good estimate of the overall average, while the average range helps estimate the standard deviation (Bakir, 2004).

4. Moving Range Control Chart

Moving range (MR) charts, also called individuals and moving range charts, are control charts that show the absolute differences between consecutive measurements of a process. They are often used alongside X-bar charts, which display the average values of the process output. These charts together help monitor both the average performance and the variability of a process (Roes et al. 1993; Rigdon et al. 1994; Amin & Ethridge, 1998).
To create a moving range chart, processed data is gathered in order and the absolute differences between each pair of consecutive observations are calculated. Then these differences are plotted on the chart. Also, the average of these

differences needs to be found, which helps set the upper control limit (UCL) and the lower control limit (LCL). The UCL is typically three times the average moving range, while the LCL is usually zero. These limits indicate the expected variation in the process (Kareem et al. 2019). When interpreting a moving range chart, scientists look for points or patterns that fall outside these control limits or suggest changes in variation. Signs of special causes for variation include a point above the UCL, a point below the LCL, or certain patterns of consecutive points above or below specific thresholds. If any of these signals are spotted, it is important to investigate and address the underlying cause. It is also worth noting that moving range charts can sometimes be misinterpreted, so understanding the process and choosing the right calculation method is crucial. In some cases, using a fixed range might be more effective than a moving one.

Overall, moving range charts are a straightforward and powerful tool for monitoring process variability in Six Sigma. They help in assessing the stability and capability of the process, estimate standard deviation, and check against customer requirements. Plus, they can highlight areas for improvement and facilitate better communication and teamwork by allowing them to share process data with other team members and stakeholders. In the end, moving range charts support ensuring that the process remains stable, capable, and continuously improving.

5. Wavelets

A wavelet is a wave-like oscillation that starts at zero, rises to a peak, and then returns to zero (Walker, 1999). It has a specific point where it is strongest, a particular oscillation period, and a scale that describes how it grows and shrinks. Wavelet analysis first appeared in the mathematics world in the 1980s and became more popular in geophysics by the 1990s.

Wavelets are handy for analysing signals, processing images, and compressing data. While keeping some sense of where things happen in time or space, wavelets help in separating information at different scales. For example, the FBI typically uses wavelets to compress and store fingerprint data (Omer et al. 2024).

Wavelets work in a way that involves modifying one or two basic waveforms, which makes them great for studying fractal fields. They are especially useful for analysing time series data that changes over time, which traditional Fourier analysis may fail to do (Ali et al. 2018).

In terms of images, wavelets can effectively compress data from satellite or radar images. As the highest frequencies are removed, the important local details are kept, resulting in a low-resolution version of the original image. On the other hand, the Fourier analysis is known to lose the recognizable features of an image if too many harmonics are removed because instead of local patterns, Fourier focuses on global ones (Ali and Mohammad, 2021).

Overall, wavelets are generally thought of as a middle ground between looking at data at specific times, where one gets detailed timing information and analysing it in frequency space, where frequency insights are gained but timing details are lost. Wavelet analysis allows us to keep some of both, making it a useful compromise.

6. Haar Wavelet

The Haar wavelet is the simplest wave among all and is considered the first known wavelet, which was proposed by Alfred Haar in 1909 and is named after the scientist. Because of its simplicity, it is usually the first choice for people who want to learn about wavelets and their specifications (Antoniadis, 2007).

To generate a Haar wavelet, consider the constraints on the h_k for N=2. The stability condition enforces $h_0 + h_1 = 2$, while the accuracy condition implies $h_0 + h_1 = 0$, and the orthogonality gives $h^2{}_0 + h^2{}_1 = 2$.

Then a unique solution exists (Gencay et al. 2002):

$h_0 = h_1 = 1$, using

$$\emptyset(x) = \emptyset(2x) + \emptyset(2x-1) \tag{1}$$

The scaling function is satisfied by a box function

$$B(x) = \begin{bmatrix} 1 & 0 \le x \le 1 \\ 0 & o.w \end{bmatrix} \tag{2}$$

Define the function ψ as

$$\psi(x) = \psi(2x) - \psi(2x-1) \tag{3}$$

Then the Haar wavelet was obtained,

$$\psi(x) = \begin{cases} 1 & 0 < x \leq \frac{1}{2} \\ -1 & \frac{1}{2} < x \leq 1 \\ 0 & o.w \end{cases} \tag{4}$$

The function $\emptyset(x)$ is the Haar scaling function and $\psi(x)$ is the Haar Wavelet.

7. Discrete Wavelet Transformation

The discrete wavelet transform (DWT) is an algorithm that processes data and is highly used in various applications, including science, mathematics, engineering, and computer science. Using a mother wavelet, which can also be referred to as a compactly supported basis function, it decomposes data and provides a multiresolution representation of the data (Ali & Awaz, 2017).

In this example where a vector of data X consists of 2^j observation and j is an integer, the DWT of X is

$$W = wX \tag{5}$$

Where W is an $n * 1$ a vector comprising both discrete scaling and wavelet coefficients. The vector of wavelet coefficients can be organized into $j + 1$ vectors.

$$W = [W1, W2, \dots, Wj_0, Vj_0]^T \tag{6}$$

Where Wj is a length $N_j = {}^{N}/_{2^j}$ vector of wavelet coefficients (Details) associated with changes on a scale of length $\lambda_j = 2^{j-1}$ symbolled as DC, and V_{jO} is a length $N_j = {}^{N}/_{2^j}$ vector of scaling coefficients (approximation or smoothing) associated with the average on a scale of length $\lambda_j = 2^{Vj_0}$ symbolled as AC, and w is an orthonormal $N \times N$ matrix associated with the orthonormal wavelet basis chosen.

The DWT coefficients for the data X can be described in a hierarchical process as follows.

After each Discrete Wavelet Transform (DWT), using the same filter as before, we break down the approximation coefficients into different bands. This allows us to combine the specifics of the latest breakdown. At each level, we can rebuild the denoised signal by using the inverse transform.

$$X = Ww^T = \sum_{j=1}^{j_0} W^T{}_j W_j + V^T{}_{j_0} V_{j_0} \tag{7}$$

The discrete wavelet transform of the Haar (or Db1) wavelet was chosen in the proposed chart configuration because it is the only linear wavelet available from the Daubechies wavelets and therefore it is a Daubechies of the first order (Db1) while the rest of the Daubechies (DbN) wavelets (N = 1, 2, …, 45) are nonlinear and do not have extension of the original observations while the nonlinear wavelets extend the number of original observations. The discrete wavelet transform of the Haar wavelet is the theoretical basis of the Daubechies wavelet which has been used in many areas of noise processing, most notably time series analysis.

8. Universal Threshold

The Universal Threshold, developed by Donoho and Johnstone, is a popular thresholding technique for wavelet transforms, particularly in denoising signals and data processing. Its underlying principle is to set a threshold to remove or suppress noise in wavelet coefficients while retaining the essential signal characteristics. This approach is based on the idea that in the wavelet transform domain, noise appears across a wide range of smaller coefficients, while the actual signal is usually concentrated in the larger coefficients.

The Universal Threshold is particularly effective because it considers the signal length and noise level. The logarithmic term increases the threshold for larger datasets, reflecting that longer signals tend to have more noise components spread across their wavelet coefficients.

9. Proposed Charts

The Haar wave partitions the data on the qualitative characteristics of the produced material into two parts. The first part represents the approximation coefficients (scale function) with n/2 coefficients and n is the sample size, which is proportional to the general average of observations of the qualitative characteristic. In contrast, the second part describes the detail coefficients (the mother wavelet function) with n/2 coefficients (DC), which are proportional to the differences (variance) of the observations of the qualitative characteristic.

The proposed charts are based on Haar wavelet analysis by creating two charts, the first for controlling the Haar approximation coefficients and the second for controlling the Haar detail coefficients, and they were as follows:

First Chart: Haar approximation coefficients Chart: The approximation coefficients (AC) are the points plotted on this chart which are obtained through the following:

Define $V_0 = x$ and x is the observations vector of length (n) and set $j = 1$ (the level) input to the j^{th} stage of the pyramid algorithm is V_{j-1} (is full-band) and related to frequencies $[0, 1/2^j]$ in x. Half-band filters for $i = 0, 1, \ldots, N_j-1$ are:

$$DC \equiv W_{j,i} = \sum_{l=0}^{L-1} h_l V_{j-1,2i+1-l \, mod \, Nj-1} \quad (8)$$

$$AC \equiv V_{j,i} = \sum_{l=0}^{L-1} g_l V_{j-1,2i+1-l \, mod \, Nj-1} \quad (9)$$

Can be placed in vectors W_j and V_j where W_j are wavelet coefficients (DC) for scale $\gamma_j = 2^{j-1}$ and V_j are scaling coefficients (AC) from equation (9) for scale $\delta_j = 2^j$. j is increased and the above is repeated until $j = J_0$ yields DWT (DC and AC) coefficients W_1, W_2, …, W_{J0}, V_{J0}. More generally, any even number is used for the number of observations by the maximum overlap discrete Haar wavelet transform (MODWT for Haar) to obtain the approximation and detail coefficients.

The target line (T_A) represents the average of the AC:

$$T_A = \sum_{i=1}^{n/2} \frac{AC_i}{n/2} \quad (10)$$

The control limits (UCL_A and LCL_A) are:

$$UCL_A = T_A + 3\theta \quad (11)$$

$$LCL_A = T_A - 3\theta \quad (12)$$

Where θ represents the threshold level estimated from the maximal overlap discrete transformation coefficients at the first level using the Universal method, that is:

$$\theta = Median\left(\left|W_{1,i} - Median\left(W_{1,i}\right)\right|\right)/0.6745 \quad (13)$$

The constant value (0.6745) represents the median of the standard normal distribution. The universal threshold level (θ) was used to replace the standard deviation of the coefficients in calculating the control limits because it is the border between noise and the true values of the observations, thus it was used as the limiting between the qualitative characteristic that conforms to the required specifications and does not conform to the required specifications, taking into account the number (10) of the standard normal distribution. and the average of those coefficients.

Second Chart: Haar Detail coefficients Chart: The Detail coefficients (DC) are the points plotted on this chart which are obtained through equation (8), The target line (T_D) represents the average of the DC:

$$T_D = \sum_{i=1}^{n/2} \frac{DC_i}{n/2} \quad (14)$$

The control limits (UCL_A and LCL_A) are:

$$UCL_D = T_D + 3\theta \quad (15)$$

$$LCL_D = T_D - 3\theta \quad (16)$$

10. Simulation Study

To illustrate the idea of the proposed charts and the points drawn on them, (6) random observations were generated, and the discrete wavelet transform for Haar wavelet was used at the first level to obtain three values for the approximation coefficients and three values for the detail coefficients, as in Figure (1).

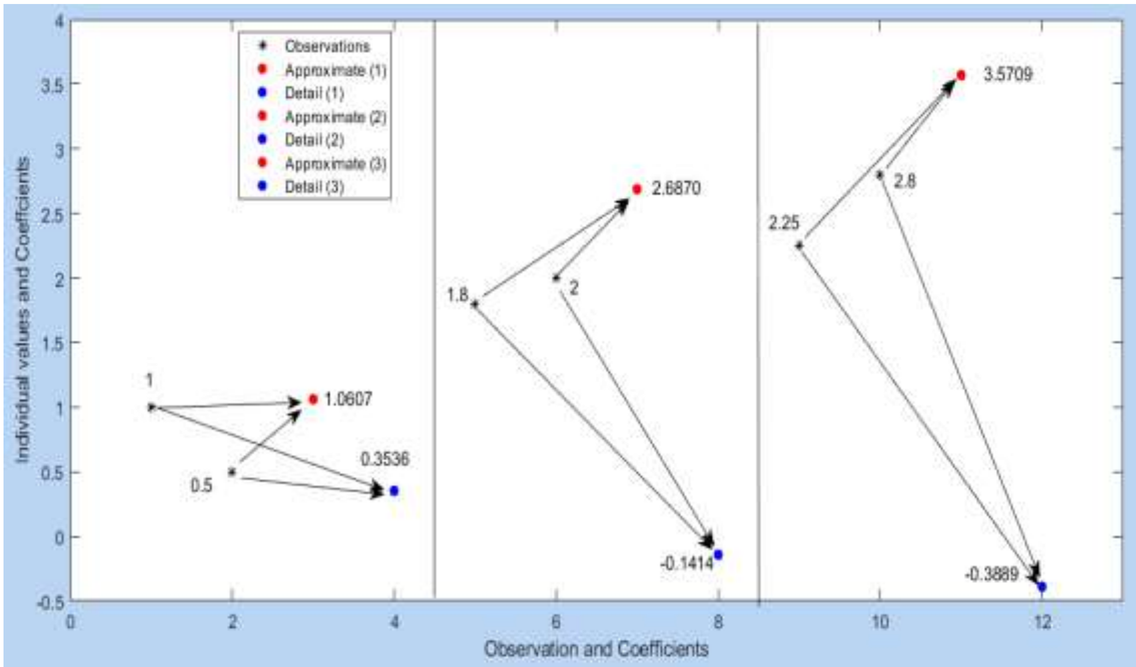

Figure 1. Observations and DWT For Haar Wavelet

Figure 1 shows that the first two observations (1 and 0.5) can be converted into two coefficients. The first represents the approximation coefficient (1.0607), which is proportional to the average of the two observations (the red point), i.e., the sum of the two observations multiplied by the Haar mother wavelet coefficient $(1/\sqrt{2})$, the second represents the detail coefficient (0.3536), which is proportional to the variance (or difference) of the two observations (the blue point), i.e., the first observation minus the second observation and then multiplied by the Haar mother wavelet coefficient $(1/\sqrt{2})$. In the same way, the third and fourth observations were segmented to obtain the second approximation and detail coefficients, and so on for the fifth and sixth observations. The approximation and detail coefficients are the points drawn on the proposed charts (the approximation and detail coefficients charts, respectively).

To compare the proposed and traditional charts, data generation from normal distribution was repeated (1000) times in simulating the Individual control chart for several different sample sizes (24, 28, and 32), The sample size chosen is well suited to single-value charts (because these charts are for controlling rare qualitative characteristics or destructive sampling). Mean values (50, 75, and 100) and variance values (1, 3, and 9) where variance equals one fits the standard normal distribution (small variance), variance equals 3 fits the traditional control limits that add and subtract three standard deviations to the target line (med variance), and finally variance (9) fits data with high variance (big variance). The proposed charts were created for the approximation and detail coefficients of the Haar wavelet and compared with traditional charts (Individual and moving average control charts) based on the minimum difference (DF = Difference between upper control limit and target line) and the average of the results are summarized in the following tables:

Table 1. Average of Results for Simulation (Mean = 50, Variance = 1)

Chart	m	UCL	LCL	Target	Variance	DF
Approximate	24	72.8075	68.6115	70.7095	1.0194	**2.0980**
Detail		2.0909	-2.1015	-0.0071	0.9687	**2.0980**
Individual		52.9591	47.0393	49.9992	0.9910	2.9599

Moving Range		3.6359	0.0000	1.1129	0.6832	2.5230
Approximate	28	72.8176	68.5941	70.7058	1.0084	**2.1117**
Detail		2.1051	-2.1184	-0.0067	0.9677	**2.1117**
Individual		52.9629	47.0303	49.9966	0.9878	2.9663
Moving Range		3.6437	0.0000	1.1153	0.6853	2.5284
Approximate	32	72.8175	68.6038	70.7106	1.0078	**2.1068**
Detail		2.1002	-2.1135	-0.0067	0.9751	**2.1068**
Individual		52.9712	47.0287	50.0000	0.9899	2.9712
Moving Range		3.6498	0.0000	1.1172	0.6927	2.5327

Table 2. Average of Results for Simulation (Mean = 75, Variance = 3)

Chart	m	UCL	LCL	Target	Variance	DF
Approximate	24	109.6978	102.4301	106.0639	3.0582	**3.6339**
Detail		3.6216	-3.6461	-0.0123	2.9062	**3.6339**
Individual		80.1252	69.8718	74.9985	2.9731	5.1267
Moving Range		6.2976	0.0000	1.9276	2.0497	4.3700
Approximate	28	109.7153	102.4000	106.0576	3.0252	**3.6576**
Detail		3.6461	-3.6692	-0.0116	2.9032	**3.6576**
Individual		80.1318	69.8564	74.9941	2.9633	5.1377
Moving Range		6.3111	0.0000	1.9318	2.0560	4.3794
Approximate	32	109.7151	102.4167	106.0659	3.0234	**3.6492**
Detail		3.6376	-3.6607	-0.0116	2.9253	**3.6492**
Individual		80.1462	69.8536	74.9999	2.9698	5.1463
Moving Range		6.3217	0.0000	1.9350	2.0780	4.3867

The three Tables (1-3) show the efficiency of the proposed charts (The charts with shorter control limit periods or DF are more sensitive to slight changes that can occur in production processes) compared to traditional charts and the proposed charts obtained a minimum difference for all cases of the sample size (24, 28, and 32), mean (50, 75, and 100), and variance (1, 3, and 9) of (2.0980, 2.1117, 2.1068, 3.6339, 3.6576, 3.6492, 6.2940, 6.3352, and 6.3205) respectively for the approximation and detail coefficients respectively of the Haar wavelet, compared to traditional charts for individual and moving average control charts. The difference between the upper control limit and target line is equal for the approximation and detail coefficients Haar wavelet charts.

Table 3. Average of Results for Simulation (Mean = 100, Variance = 9)

Chart	m	UCL	LCL	Target	Variance	DF
Approximate	24	147.7118	135.1238	141.4178	9.1746	**6.2940**
Detail		6.2728	-6.3152	-0.0212	8.7185	**6.2940**
Individual		108.8772	91.1178	99.9975	8.9193	8.8797
Moving Range		10.9077	0.0000	3.3388	6.1491	7.5690
Approximate	28	147.7421	135.0716	141.4069	9.0756	**6.3352**
Detail		6.3152	-6.3553	-0.0200	8.7095	**6.3352**
Individual		108.8886	91.0909	99.9897	8.8898	8.8988
Moving Range		10.9312	0.0000	3.3460	6.1679	7.5853
Approximate	32	147.7417	135.1006	141.4211	9.0702	**6.3205**
Detail		6.3005	-6.3406	-0.0200	8.7758	**6.3205**
Individual		108.9136	91.0861	99.9999	8.9095	8.9137
Moving Range		10.9495	0.0000	3.3516	6.2341	7.5980

The approximation and detail coefficients variance are very close to the variance assumed in the simulation and the variance of individual observations. It becomes more accurate as sample sizes increase. The target line for the approximation coefficients chart is larger than the target line for the individual observations and moving average charts. In contrast, the target line for the detail coefficients chart is close to zero and is equal to zero if there is no noise in the data according to the properties of wavelet analysis.

11. Real Data

A bank's mortgage loan processing unit "monitors the costs of processing loan applications. The quantity tracked is the average weekly processing costs, obtained by dividing total weekly costs by the number of loans processed during the week." (Montgomery, 2011).

Phase I Operation and Interpretation of the Charts.
Appendix (Table I) shows the processing costs in the last 20 weeks whereas Figure 2 highlights the individual that is set up along with the moving range control charts for the data.

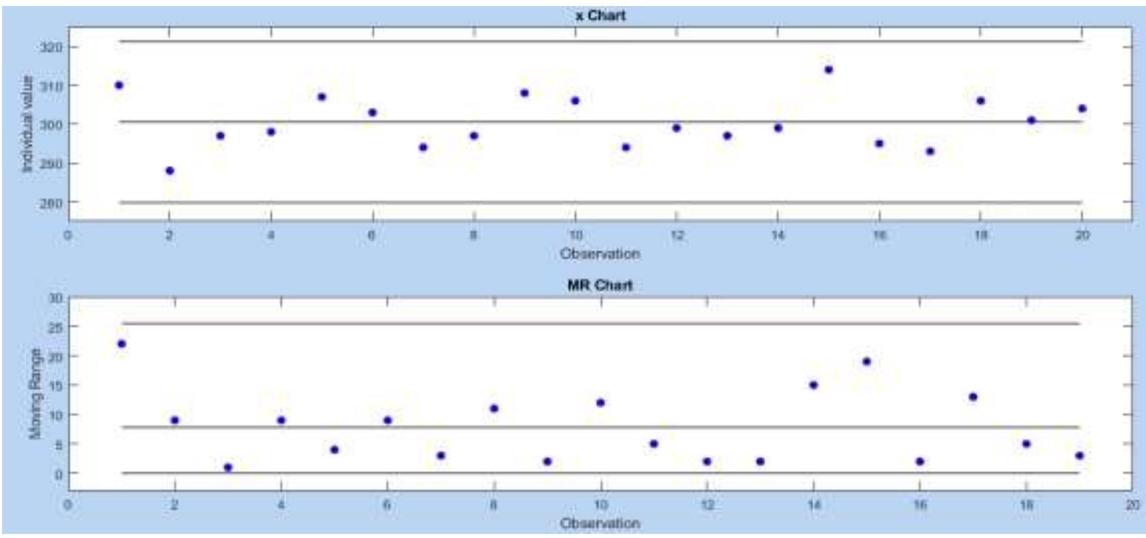

Figure 2. Individual and Moving Range Charts for Real Data (Phase I)

Figure 2 of the classical charts for the first time (Phase I) shows that all points are within the control limits. This means that these charts can be used to control qualitative characteristics (the average weekly processing costs) in future (Phase II).
Figure 3 of the approximation coefficients chart for the first time (Phase I) shows that all points are within the control limits. This means that this chart can be used to control qualitative characteristics for approximation coefficients in future (Phase II). The detail coefficients chart shows that there is one point that is outside the control limits, so the kill (the coefficient equal to zero) or keep rule was used, and then the detail coefficients chart was created as in the following figure:

Figure 4 of the modified proposed charts for the first time (Phase I) shows that all points are within the control limits. This means that these charts can be used to control qualitative characteristics for approximation and detail coefficients in future (Phase II).

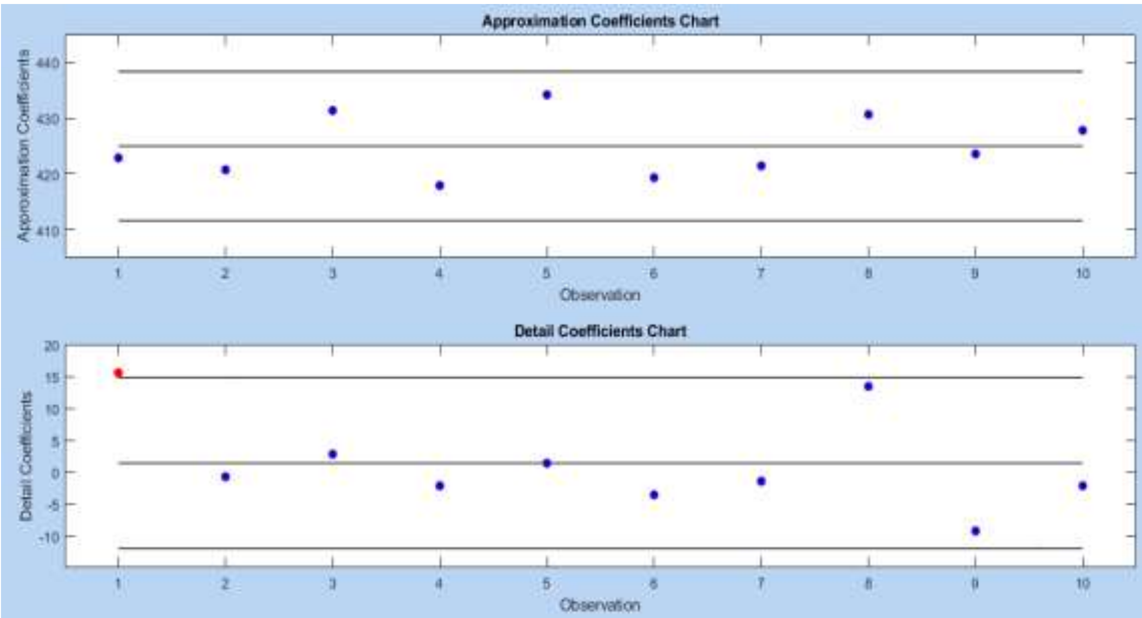

Figure 3. Approximation and Detail Coefficients Charts for Real Data (Phase I)

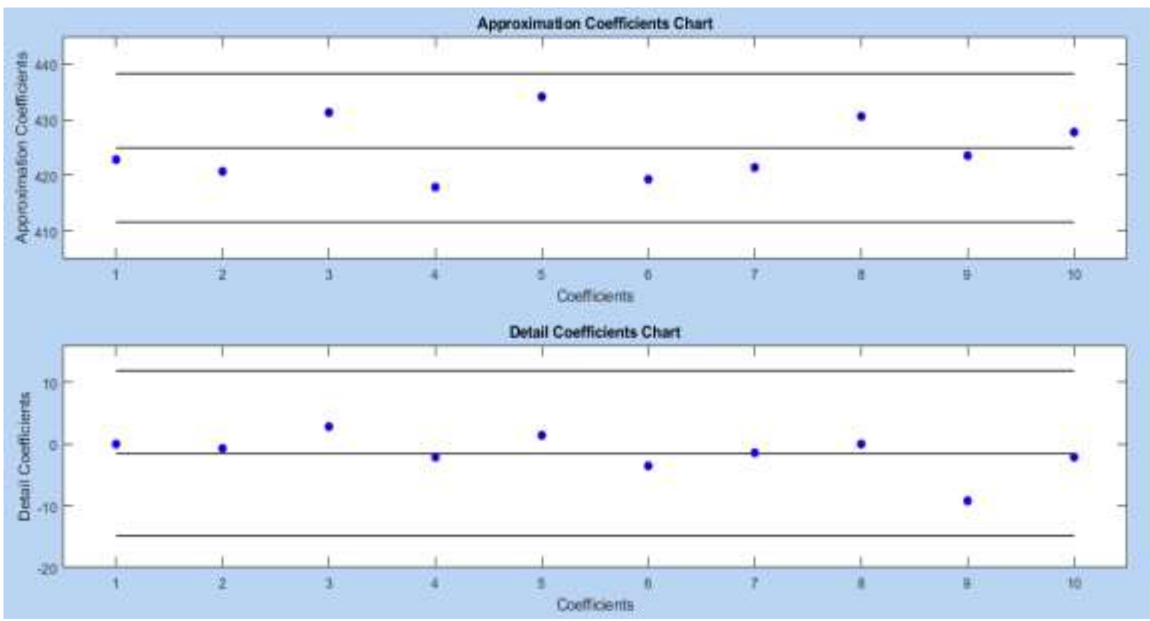

Figure 4. Modified Approximation and Detail Coefficients Charts for Real Data (Phase I)

Table 4. Results for Real Data

Chart	UCL	LCL	Target	Variance	DF
Approximate	438.3144	411.6280	424.9712	31.5556	**13.3432**
Detail	11.8583	-14.8281	-1.4849	10.7167	**13.3432**
Individual	321.2167	279.7833	300.500	43.4211	20.7167
Moving Range	25.4482	0.0000	7.7895	38.6199	17.6587

Tables (4) show the efficiency of the proposed charts compared to traditional charts, and the proposed charts obtained a minimum difference of (13.3432) for the approximation and detail coefficients of the Haar wavelet, compared to the individual and moving average control charts. The difference between the upper control limit and target line is equal for the approximation and detail coefficients Haar wavelet charts. A variance of the approximation and detail coefficients is less than the variances of individual observations and moving averages, and this clarifies the efficiency of wavelet analysis and the reduction of data noise. The target line for the approximation coefficients chart is larger than the target line for the individual observations chart. In contrast, the target line for the detail coefficients chart is not equal to zero due to data noise.

Phase II Operation and Interpretation of the Charts.

Table II in the Appendix shows the costs of processing mortgage applications from weeks 21 to 40. It is seen that this data visualized in Figure 4, along with the moving range control chart created in Phase I. Figure 5 shows that around week 39, there was a noticeable rise in costs that is indicated by a "shift in process level" and is followed by an out-of-control signal the next week. The moving range chart reflects this shift with a big spike at week 39, which helps in identifying the time when the average cost changed. To understand why this shift took place, one should investigate factors around week 39. Possible reasons could include temporary staff replacements due to vacations or an influx of applications needing extra manual review (underwriting work). It is important to interpret the moving range chart carefully as the data points are linked and might show patterns or cycles.

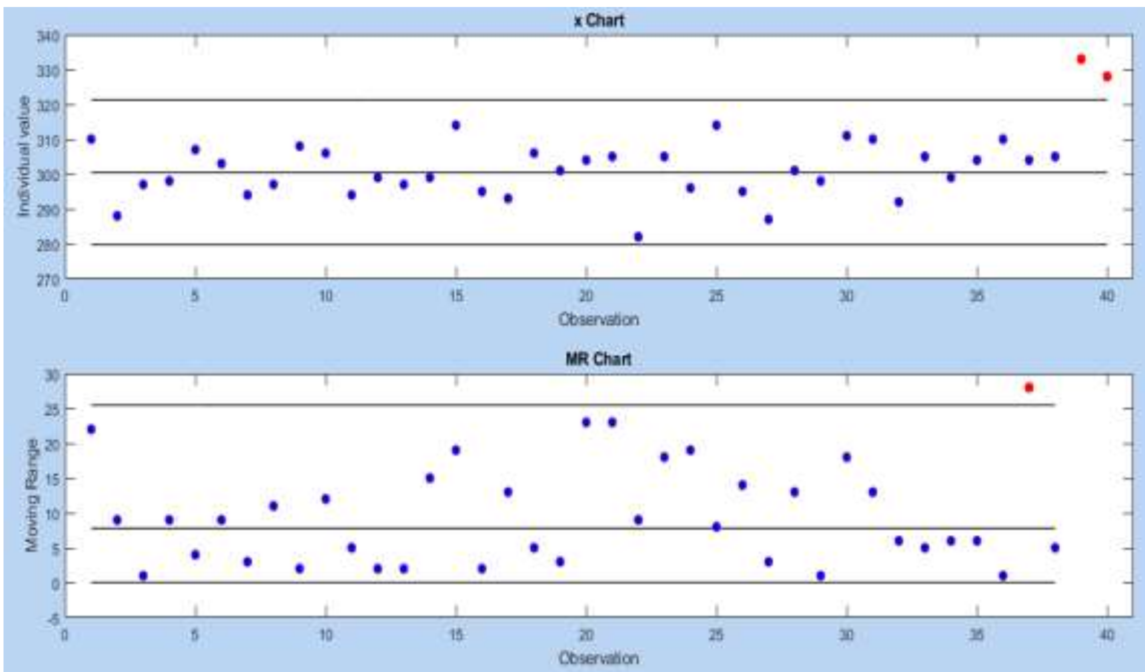

Figure 5. Individual and Moving Range Charts for Real Data (Phase II)

The charts proposed in Phase I were used to monitor the process in the following twenty weeks, as in Figure 6:

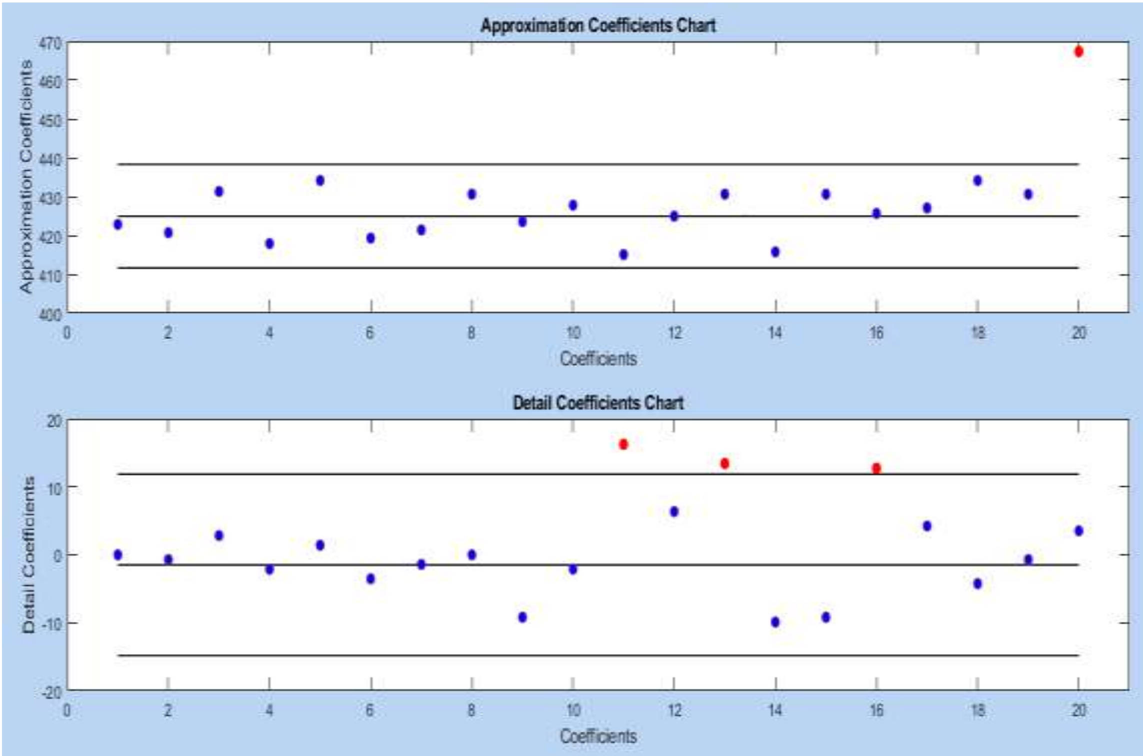

Figure 6. Approximation and Detail Coefficients Charts for Real Data (Phase II)

Each point drawn on the proposed charts represents two observations (Figure 6). The approximation coefficients chart showed that there is one point (weeks 39 and 40) out of control, while the detail coefficients chart showed three points (weeks 21, 22; 25, 26; and 31, 32) outside the control limits, and this means there is a defect in the production process.

The approximation coefficients chart revealed defects in the production process like the individual observations chart, and the detail coefficients chart revealed early defects in the production process (This can be seen from the difference between the value of weeks 21 and 22 and so on for the rest of the individual observations chart), this means the sensitivity of the chart in detecting minor changes that could occur in the production process, especially in the variance of observations, which was not specified by traditional charts (Individual observation and Moving average charts).

12. Conclusions

1. Differences (or variance) in the quality of the produced material can be controlled and monitored through the proposed detail coefficients chart, which is not available in traditional charts for controlling individual observations.

2. The proposed charts were more efficient than traditional charts depending on the difference between the control limit that is upper, and the target line and the accuracy of the variance used.

3. The proposed charts were more sensitive than traditional charts in detecting subtle changes that may occur in the production process.

4. The proposed charts composed for the first time addressed the problem of data noise by using wavelet estimation which is more accurate than traditional charts.

5. The approximation and detail coefficients charts of the Haar wavelet have equal efficiency due to the equal difference between the upper control limit and the target line is equal.

6. There is no significant effect on the accuracy of constructing the proposed and traditional charts for the first time when the number of observations increases. In contrast, this accuracy decreases with increasing variance.

7. For real data the approximation and detail coefficients and classical charts revealed defects in A bank's mortgage loan processing unit (monitors the costs of processing loan applications).

13. Recommendations

1. Using the DWT coefficients for Haar wavelet in creating quality control charts to monitor the individual observations (corresponding to the Individual observations and the moving average charts) and controlling and monitoring the differences (or variance) in the produced material, which is not available in traditional panels for Individual observations.
2. Conducting other studies to create charts of discrete wavelet transform coefficients for Daubechies wavelets, Coiflets, Symlets, etc.
3. Conducting other studies to create charts of maximal overlap discrete wavelet transform coefficients.

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Conflict of interest

The author has no conflict of interest.

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Appendix

Table I. Costs of Processing Mortgage Loan Applications

Weeks	Cost x	Moving Range MR
1	310	
2	288	22
3	297	9
4	298	1
5	307	9
6	303	4
7	294	9
8	297	3
9	308	11
10	306	2
11	294	12
12	299	5
13	297	2
14	299	2
15	314	15
16	295	19
17	293	2
18	306	13

19	301	5
20	304	3
	$\bar{X} = 300.5$	MR=7.79

Table II. Costs of Processing Mortgage Loan Applications, Weeks 21-40

Week	Cost x	Weeks	Cost
21	305	31	310
22	282	32	292
23	305	33	305
34	296	34	299
25	314	35	304
26	295	36	310
27	287	37	304
28	301	38	305
29	298	39	333
30	311	40	328

لوحات مراقبة الجودة المقترحة باستخدام معاملات مويجة هار لتحسين مراقبة الإنتاج

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الخلاصة: تتمثل إحدى المشكلات الرئيسية للوحات مراقبة الجودة التقليدية، مثل لوحة المشاهدات المفردة ولوحة المتوسط المتحرك، في أنها لا تركز على مراقبة الاختلافات في المواد المنتجة. لمعالجة هذه المشكلة، اقترح الباحثون إنشاء لوحات جديدة تعتمد على مويجة هار والتي يمكن أن تركز بشكل أكبر وتتعامل بشكل أفضل مع ضوضائية البيانات التي تؤثر على دقة اللوحات التقليدية. تستند اللوحات المقترحة الجديدة إلى طريقة التحويل المويجي المتقطع لمويجة هار. الأولى تتناسب مع متوسط المشاهدات (المعاملات التقريبية أو مرشح التمرير المنخفض) بينما تراقب اللوحة الأخرى الاختلافات بين هذه المشاهدات (معاملات التفاصيل أو مرشح التمرير العالي). لأول مرة، تم استخدام طريقة قطع العتبة الشاملة لمعالجة ضوضائية البيانات لإنشاء حدود التحكم في اللوحات المقترحة. استخدم الباحثون كل من بيانات المحاكاة والحقيقية لتطوير هذه اللوحات باستخدام برنامج MATLAB. أثبتت النتائج دقة وكفاءة اللوحات المقترحة ونجاحها في التعامل مع ضوضائية البيانات وحساسيتها في اكتشاف التغييرات الطفيفة التي قد تحدث في عملية الإنتاج.

الكلمات المفتاحية: لوحات السيطرة النوعية، التحويل المويجي المتقطع، المويجة هار، قطع العتبة الشاملة ولوحة السيطرة المفردة.